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THE BEST FAILURE CRITERIA WITH AN ANALYTICAL MODEL FOR UNDERBALANCED DRILLING

УЛУЧШЕННАЯ АНАЛИТИЧЕСКАЯ МОДЕЛЬ КРИТЕРИЕВ ОТКАЗА, ИСПОЛЬЗУЕМАЯ ПРИ БУРЕНИИ НА ДЕПРЕССИИ ПЛАСТА

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Abstract. One of the important issues in borehole instability assessments is choosing proper failure criteria. In this paper, four different failure criteria are used. These failure criteria are Mohr-Coulomb, Hoek-Brown, Modified Lade and Mogi-Coulomb failure criterion.

A poroelastic model is used for calculating collapse pressure. A MATLAB code is generated for this purpose. These failure criteria are presented in the following sections briefly. The two first failure criteria ignore the effect of intermediate stress but two other failure criteria are polyaxial failure criteria and take into account the effect of intermediate stress. Collapse pressures for different inclination and azimuth which are calculated with different failure criteria are presented here.

It is concluded that the failure criteria which ignore intermediate principle stress causes to overestimate collapse pressure. Modified Lade and Mogi-Coulomb criteria give the more realistic collapse pressures.

Аннотация. Одним из важных вопросов в оценке нестабильности скважины является выбор надлежащих критериев отказа. В данной работе, используются четыре различных критерия отказа. Это критерии отказа: Мора-Кулона, Хука-Брауна, изменения Ладе и критерий разрушения Моги-Кулона. Пороупругая модель используется для расчета коллапса давления. Для этой цели генерируется код MATLAB.

Авторами дано краткое описание перечисленных критериев отказа. Первые два критерия отказа не учитывают влияния промежуточного напряжения, а два других критерия недостаточности, являясь полиаксиальными критериями отказа, учитывают эффект промежуточного напряжения. В статье приводятся показатели

давлений для различных наклонов и азимута, рассчитанных, исходя из разных категорий критериев отказа.

По приведенным данным сделан вывод о том, что критерии, игнорирующие принцип промежуточного стресса, вызывают переоценку давления коллапса. Изменения в критериях Ладе и Моги-Кулона дают более реалистичное давление коллапса.

Keywords: failure criteria, analytical model, poroelastic model.

Ключевые слова: критерии отказа, аналитическая модель, модель пористо-упругости.

Introduction

After calculating stresses around the borehole, these stresses should be compared with formation strength. Borehole can be failed if stresses exceed the rock compressive strength. Predicting failure needs choosing proper failure criterion. There are different failure criteria for shear failure. In this paper, four different failure criteria are discussed and compared. These failure criteria are presented in the following sections briefly. The two first failure criteria ignore the effect of intermediate stress but two other failure criteria are polyaxial failure criteria and take into account the effect of intermediate stress.

Mohr-Coulomb criterion

The simplest and most important criterion is introduced by Coulomb in 1776. The mathematical form of this criterion expressed as follows:

$$\tau = c + \sigma_n \tan \emptyset \tag{1}$$

Where τ is shear stress, σ_n is normal stress, c is cohesion of rock and ϕ is internal friction angle. This criterion can be represented in terms of principle stresses. The relation between τ and σ_n and principle stresses are brought here:

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta \tag{2}$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta \tag{3}$$

θ gives the orientation of failure plane(figure1).

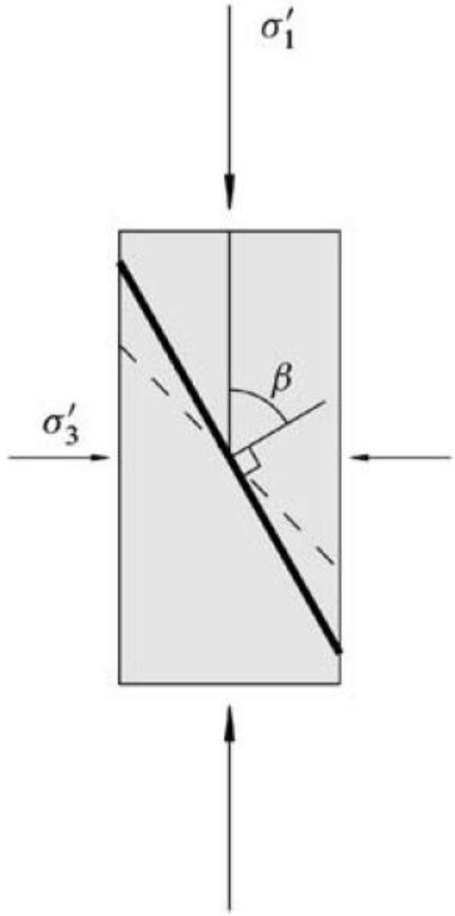

Figure1. Angle between failure plane and maximum principle stress direction (Fjear et al 1992)

Replacing equations 2 and 3 in 1gives:

$$\sigma_1 = C_0 + q\,\sigma_3 \tag{4}$$

Where:

$$C_0 = 2c\,\frac{\cos\varphi}{1-\sin\emptyset} \tag{5}$$

$$q = \frac{1+\sin\emptyset}{1-\sin\emptyset} \tag{6}$$

Also θ and ϕ are related by:

$$\varphi + \frac{\pi}{2} = 2\,\beta \quad \rightarrow \quad \beta = \frac{\pi}{4} + \frac{\emptyset}{2} \tag{7}$$

This last relation can be obtained easily from the following figure:

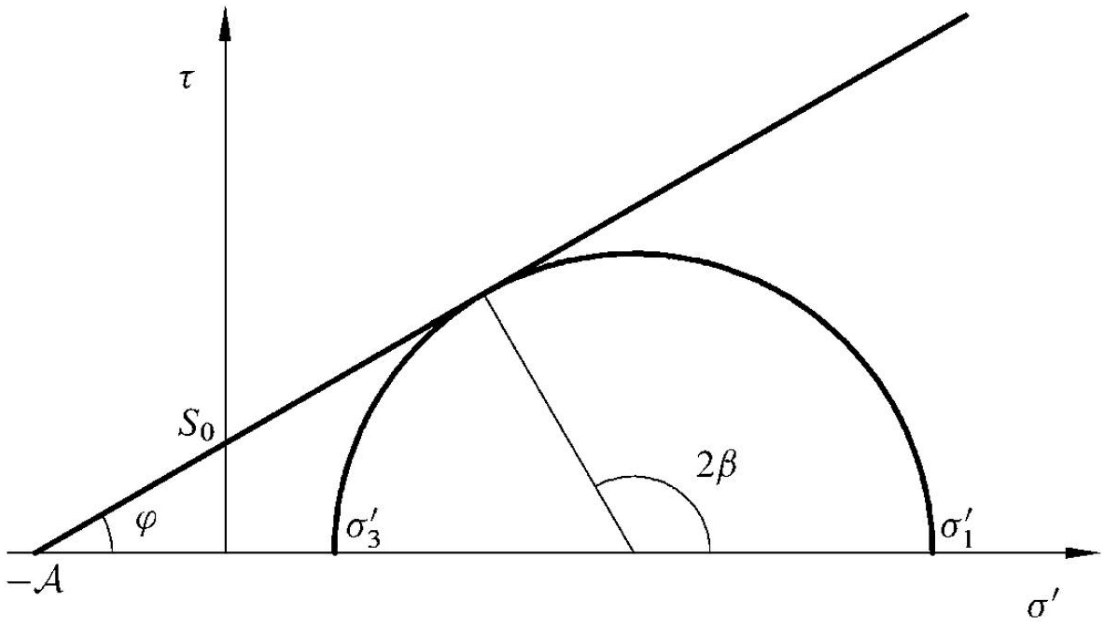

Figure 2. Mohr-Coulomb criterion in τ, $\boldsymbol{\sigma'}$ (Fjear et al 1992)

Hoek-Brown criterion

This is a completely experimental criterion based on curve fitting of triaxial test data. The conceptual starting point for the criterion was the Griffith theory for brittle fracture but the process of driving the criterion was one of pure trial and error. The original Hoek-Brown criterion was introduced in 1980 (Hoek and Brown., 1980) is as follows:

$$\sigma_1 = \sigma_3 + \sqrt{m\,\sigma_3 C_0 + s\,{C_0}^2} \tag{8}$$

$C_0$is uniaxial compressive strength of the intact rock. *m* and *s* are material constants, *s* takes the value 1 for intact rock, and less than unity for disturbed rock (Hoek and Brown, 1997). The values for *m* are different from rock to rock, with a range between about 1.4 and 40.7 (Sheorey, 1997).

Modified Lade criterion

This section presents the equations used in modified lade criterion which can be used in stability and sanding prediction. This criterion is discussed in details in Ewy (1999). The mathematical description is:

$$\frac{(I_1'')^2}{I_3''} = 27 + \eta \quad (9)$$

Where I_1'' and I_3'' are modified first and third stress invariants which defined as follows:

$$I_1'' = (\sigma_1 + S_1 - p_p) + (\sigma_2 + S_1 - p_p) + (\sigma_3 + S_1 - p_p) \quad (10)$$

$$I_3'' = (\sigma_1 + S_1 - p_p)(\sigma_2 + S_1 - p_p)(\sigma_3 + S_1 - p_p) \quad (11)$$

Where σ_1 ,σ_2 and σ_3 are three principle stresses, p_p is pore pressure and S_1 and η are materials constant. The parameter $S1$ is related to the cohesion of the rock, while the parameter η represents the internal friction, which can be calculated directly from the Mohr-Coulomb friction angle, ϕ, with the following equation (Ewy et al., 2001):

$$\eta = 4\ (\tan\emptyset)^2 \frac{9-7\ \sin\emptyset}{1-\sin\emptyset} \quad (12)$$

And S_1 is:

$$S_1 = \frac{c}{\tan\emptyset} \quad (13)$$

In two last formulas, c and ϕ are cohesion and internal friction angle respectively.

Mogi-Coulomb criterion

This criterion is presented by Al-Ajmi and Zimmerman in 2006. Their study was based on Mogi criterion (1971) which is the first polyaxial criterion. As Mogi criterion represented, the brittle fractures occur along a plane striking in the σ_2 direction and he concluded that the mean normal stress that opposes the creation of the fracture plane is $\sigma_{m,2}$(Al-Ajmi and Zimmerman, 2006):

$$\tau_{oct} = f(\sigma_{m,2}) \quad (14)$$

Where f is monotonically increasing function and τ_{oct} and $\sigma_{m,2}$ are defined as follows:

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} \quad (15)$$

$$\sigma_{m,2} = \frac{\sigma_1 + \sigma_3}{2} \quad (16)$$

In fact, Al-Ajmi and Zimmerman criterion is the linear form of Mogi-Coulomb criterion. The linear form of Mogi function is represented in the following equation:

$$\tau_{oct} = a + b\,\sigma_{m,2} \tag{17}$$

Where a and b are:

$$a = \frac{2\sqrt{2}}{3}\,c\,cos\phi \tag{18}$$

$$b = \frac{2\sqrt{2}}{3}\,sin\phi \tag{19}$$

The strength parameter b essentially represents the internal friction angle, while the parameter is related to the cohesion and the internal friction angle (Al-Ajmi and Zimmerman, 2006).

Input data for modeling

Table 1 shows all input data and approaches to obtaining them for analytical modeling.

Table 1. Input parameters for analytical model

Input data	parameters	Approaches to obtaining data
Orientation	Azimuth Inclination Depth	Surveying logs
In-situ stresses	Overburden pressure Maximum horizontal in-situ stress Minimum horizontal in-situ stress	Density logs Inversion techniques
Rock mechanic parameters	Cohesion Internal friction angle Biot's constant	Sonic logs + Laboratory tests
Pore pressure	Pore pressure	Well program or correlations

Poroelastic modeling for calculating collapse pressure

As the first step, the global in-situ stresses should be converted to stresses in local coordinate system whose z axis is parallel the wellbore axis. The following formula can be used for this purpose (Fjear, et al., 1996):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = A \times \begin{Bmatrix} \sigma_H \\ \sigma_h \\ \sigma_v \end{Bmatrix} \times A'$$

(20)

$$\mathrm{A} = \begin{bmatrix} cos\phi\ cosi & sin\phi\ cosi & -sini \\ -sin\phi & cos\phi & 0 \\ cos\phi\ sini & sin\phi\ sini & cosi \end{bmatrix}$$

Where ϕ is azimuth and i is inclination. It should be noted that azimuth is the angle between the borehole direction and maximum horizontal in-situ stress. Sometimes this angle is measured respect to the north which should be corrected.

The second step in calculations is converting the local stresses to cylindrical form. These stresses are the sum of the stresses at wellbore wall$(r = r_w)$ and local stress and in-situ stresses and hydraulic effects.

$$\sigma_{rr} = p_w$$

$$\sigma_{\theta\theta} = (\sigma_x + \sigma_y) - 2(\sigma_x - \sigma_y)\cos 2\theta - 4\tau_{xy}\sin 2\theta - p_w + \frac{\alpha(1-2\nu)}{1-\nu}(p_w - p_i) \quad (21)$$

$$\sigma_{zz} = \sigma_z - \nu\,[2(\sigma_x - \sigma_y)\cos 2\theta + 4\tau_{xy}\sin 2\theta\,] - \frac{\alpha(1-2\nu)}{1-\nu}(p_w - p_i)$$

$$\tau_{\theta z} = 2(\tau_{yz}\cos\theta - \tau_{xz}\sin\theta)$$

$$\tau_{r\theta} = \tau_{rz} = 0$$

θ is somewhere around the borehole where fracture initiates. This angle is unknown. For calculating θ, maximum stress acting on wellbore wall and its location should be calculated.

Actually both normal stresses, tangential stress and axial stress, will reach the maximum and minimum values at the same points. Taking first derivative from both, gives same results. By taking the first derivative from σ_θ with respect to θ, the locations of maximum and minimum are as follows (Al-Ajmi and Zimmerman, 2006):

$$\frac{\partial\sigma_\theta}{\partial\theta} = 4(\sigma_x - \sigma_y)\sin 2\theta - 8\tau_{xy}\cos 2\theta$$

(22)

$$\frac{\partial\sigma_\theta}{\partial\theta} = 0 \xrightarrow{yields} \theta_1 = Arctan\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right), \theta_2 = \theta_1 - \frac{\pi}{2}$$

These two angles should be placing in σ_θ to find the maximum and minimum values but p_w is unknown. The last two terms in σ_θ have the same magnitude with both angles so they could be ignored and new term is called $\sigma_{\theta d}$ (Al-Ajmi and Zimmerman, 2006):

$$\sigma_{\theta d} = (\sigma_x + \sigma_y) - 2(\sigma_x - \sigma_y)\cos 2\theta - 4\tau_{xy}\sin 2\theta, \theta = \theta_1, \theta_2 \tag{23}$$

This should be noted that θ_1 in equation 22 could be obtained from slope of eigen vectors of following matrix which all of arrays are calculated in the first step:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \tag{24}$$

For using criteria, principle stresses should be calculated. The radial stress is one of principle stresses because there is no shear stress in this direction. But there is shear stress in the direction of tangential and axial stresses. So principle stresses in these directions should be calculated. Again eigenvector and eigenvalue concept is used to calculate the principle stresses.

Then the three principle stresses are compared to each other to find maximum and minimum principle stress to use in failure criterion. In this program four different criteria are used (Mohr-Coulomb, Hoek-Brown, Modified Lade and Mogi-Coulomb).

Mohr-Coulomb criterion is brought here for example. The program continues until the user defined function matches with the predefined condition.

Mohr-Coulomb criterion is:

$$\sigma_1 - p_p = C + q\,\sigma_3 \tag{25}$$

And condition for stopping the program is:

$$F = \sigma_1 - p_p - C - q(\sigma_3 - p_p) \geq 0 \tag{26}$$

Comparing different failure criteria

In this paper, four different failure criteria are compared to find the criterion which predicts more realistic collapse pressure. These four failure criteria are: Mohr-Coulomb criterion, Hoek-Brown criterion, Modified Lade criterion and Mogi-Coulomb criterion. It is known from the literature that Mohr-Coulomb predicts overly conservative outcome of collapse density. Hoek-Brown criterion is more exact than Mohr-Coulomb criterion but it predicts conservative collapse pressure too. The conservative results for these two failure criteria are due to ignoring the intermediate stress (Single et al., 1998, Ewy 2001 and Kristiansen 2004).

For the Modified Lade and Mogi-Coulomb criteria, the conditions are different.

These are 3D failure criteria. 2D failure criteria predict higher mud weight to prevent collapse. So a criterion should be used which predicts collapse pressure lower than Mohr-Coulomb and Hoek-Brown criteria. Here new failure criterion, Mogi-Coulomb criterion, is used.

For comparing different failure criteria, table 2 is used.

Table 2.Input data for comparing different failure criteria

Vertical stress, psi	Max. horizontal stress, psi	Min. horizontal stress, psi	Biot's constant	Reservoir pressure	Friction angle, degrees	Cohesion, pressure	Poisson's ratio
4000	3500	3000	0.9	2000	30	1000	0.29

Two cases are assumed. For the first case, azimuth is taken to be constant for different inclinations and collapse pressures for four different criteria are calculated and plotted. In the second case, in a constant inclination, collapse pressures are calculated for different azimuths. The results are shown in the following figures.

Figure 3 shows the variation of collapse pressure with inclination in a constant azimuth. Here azimuth is assumed to be equal to 20 degrees. Figure 4 shows the variation of collapse pressure with azimuth in constant inclination. Here inclination is assumed to be 90 degrees.

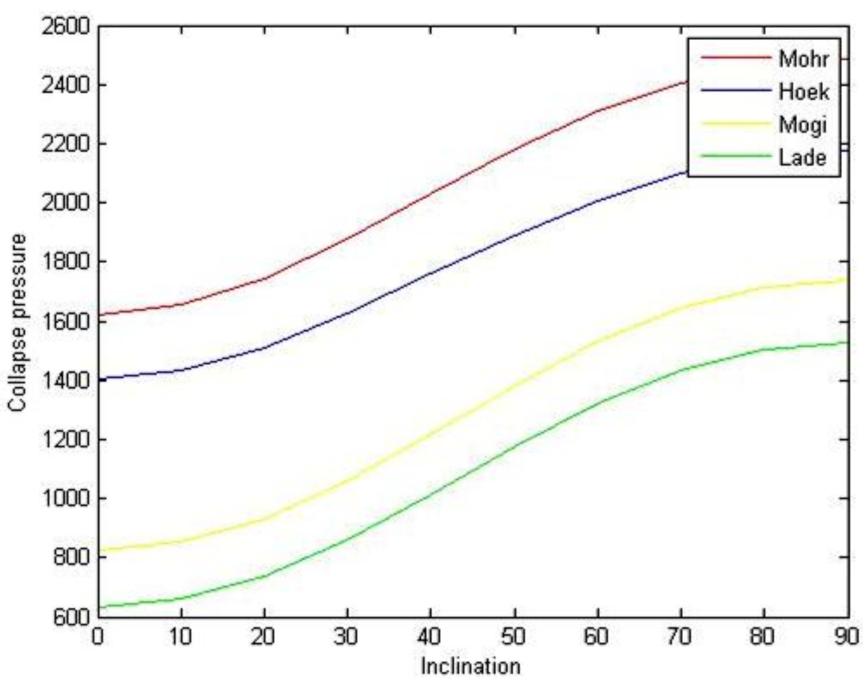

Figure 3. Variation of collapse pressure with inclination in constant azimuth

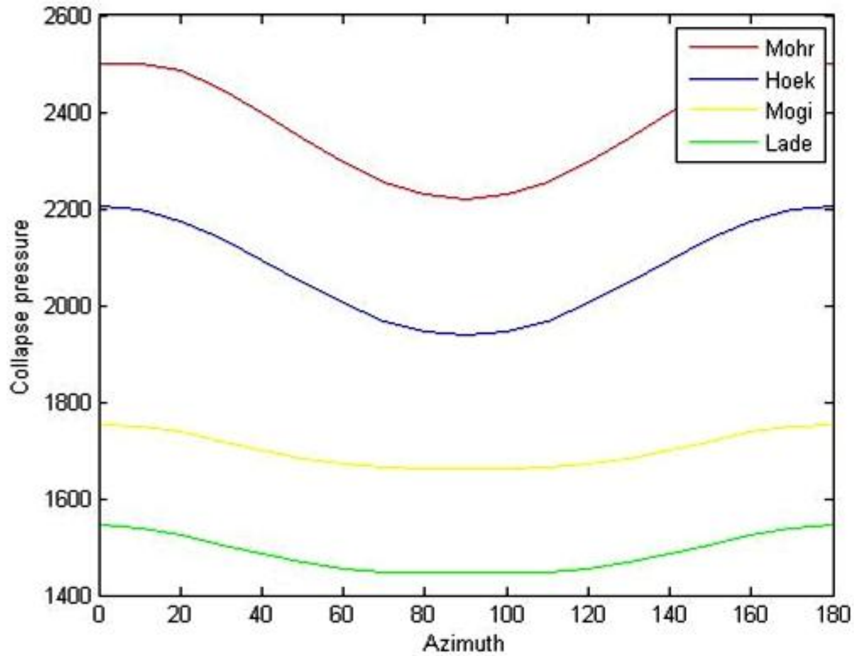

Figure 4. Variation of collapse pressure with azimuth in constant inclination

Conclusions

In this paper, four different failure criteria are compared with each other. A poroelactic model is assumed. For this purpose, a MATLAB code is generated. For any failure criterion, collapse pressures are calculated for different inclinations and azimuths. The results are illustrated in this paper with two figures. It is concluded that the failure criteria which ignore intermediate principle stress predict higher collapse pressure. Also it is concluded that 3D failure criteria such as Modified Lade and Mogi-Coulomb failure criterion give more realistic collapse pressure due to taking intermediate principle stress into account.

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