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ANALYTICAL MODEL OF TEMPERATURE PREDICTION FOR A GIVEN PRODUCTION HISTORY

A.Sh. Ramazanov, V.M. Nagimov, R.K. Akhmetov
Bashkir State University, Ufa, Russia
e-mail: ramaz@geotec.ru

Abstract. The mathematic model was developed for temperature calculation for two cases: with/without radial heat conduction. Joule-Thomson effect, thermal expansion or compression, heat conduction were taken into account in the temperature model. Computation algorithm was designed for temperature prediction in case of a given production history. Computer software was developed on the basis of described computation algorithm. Both computation algorithm and computer software were tested using simple analytical models.

Keywords: Temperature prediction, multi rate test, analytical model, Joule-Thomson effect, adiabatic effect, thermal expansion, baro-thermal effect, heat conduction, characteristics method, inverse problem.

Introduction. Thermometry is becoming more quantitative tool in recent years [1, 2, 3] along with the using of temperature logging as a main method for integrity evaluation [4, 5]. Obviously, the numerical modeling is playing a major role in quantification of temperature logging data. However, simple analytical methods for temperature prediction are still taking an important place due to at least two applications. First, the inverse problem could be effectively solved using temperature data on the basis on analytical solution. Second, complex numerical models could be tested using simple analytical equations.

Up to the moment analytical models for temperature prediction are available for certain cases including temperature calculation for given either wellbore pressure changing over time [6] or constant (fixed) downhole flowrate [7]. Nowadays, Multi Rate Testing is becoming very popular as an advanced well testing method [8]. This paper describes an analytical model for temperature prediction for a given production history, for example, in case of Multi Rate Test.

1. Analytical Temperature model with no heat conduction

Consider we have single-phase fluid flow through the porous media. Temperature perturbations in the reservoir are caused by Joule-Thomson effect, thermal expansion or compression (adiabatic effect) and heat conduction. Initially, we have a disturbed temperature profile within the reservoir. The part of the temperature perturbations due to pressure changing in the reservoir is known as barothermal effect [9].

Mathematical Statement. Here we have the mathematical model including the following:

- horizontal homogeneous porous reservoir;
- initial temperature field given as a function $f(r)$;
- heat exchange between the reservoir and overburdens strata is neglected (adiabatic assumption);
- radial heat conduction is neglected;
- single phase fluid flow in the reservoir.

We have to consider the following heat transfer equation for described physical model to calculate the temperature perturbations [7]:

$$\frac{\partial T}{\partial t}+u(r,t)\frac{\partial T}{\partial r}=-\varepsilon u(r,t)\frac{\partial p(r,t)}{\partial t}+\eta\Omega\frac{\partial p(r,t)}{\partial t} \tag{1}$$

$$T(r,0)=f(r). \tag{2}$$

Here:

T(r,t) – temperature, *K*;

p(r,t) – pressure field, *Pa*;

ε – Joule-Thomson coefficient, *K/Pa*;

η – adiabatic coefficient of expansion/compression, *K/Pa*;

$u(r,t)=\bar{c}\,\vartheta(r,t)=-\bar{c}\,\frac{k}{\mu}\frac{\partial p}{\partial r}$ – convection velocity, m/s;

k - permeability, m^2;

μ - viscosity, $Pa\cdot\sec$;

$\Omega=\phi\bar{c}$;

ϕ - porosity, frac;

$\bar{c}=\frac{c_1\rho_1}{C_{for}}$ – a number obtained from specific fluid heat capacity divided by specific formation heat capacity saturated by the fluid.

Ω – a special parameter which is responsible for simultaneous heat exchange between fluid and reservoir matrix.

Note that right-hand side of equation (1) as a source term is a mathematical description of barothermal effect.

General solution of equation (1) could be obtained for any given pressure distribution $p(r,t)$ by method of characteristics which are determined by functions $r_t = r(t, r_1)$. These functions are calculated from a system below:

$$\begin{cases} \dfrac{dr}{dt} = u(r,t), \\ r(0) = r_1; \end{cases} \tag{3}$$

Thus, general solution of equation (1) could be presented as the following [6]

$$T(r(t,r_1),t) = f(r_1) + \varepsilon\left(p(r_1,0) - p(r_t,t)\right) + (\varepsilon + \eta\Pi)\int_0^t \frac{\partial_2 p(r_\tau,\tau)}{\partial\tau}\,d\tau \quad . \tag{4}$$

If we have any certain reservoir pressure distribution over time $p(r,t)$ than the temperature field could be fully predicted within the reservoir by equation (4).

Temperature prediction for a given production hystory[1]. If we consequently know the production history i.e. we have a function $Q(t)$ we can reproduce the pressure distribution over time by the formula described in [10, 11]:

$$p(r,t) = P_i + \frac{Q_1\mu}{4\pi kh}\left[E_i\left(-\frac{r^2}{4\chi t}\right) + \sum_{i=2}^{N}\frac{Q_i - Q_{i-1}}{Q_1}E_i\left(-\frac{r^2}{4\chi(t - t_{i-1})}\right)\right], \tag{5}$$

Here:

Q_i – the constant (fixed) flow rate in the time range $t_{i-1} < t < t_i$, m^3/sec;

h – working thickness, *m*;

$E_i(-x)$ – exponential integral determined from the following:

$$E_i(-x) = -\int_x^\infty \frac{e^{-\xi}}{\xi}\,d\xi \tag{6}$$

Specific convection flow velocity is obtained from the linear Darcy's equation:

$$u(r,t) = -\bar{c}\,\frac{k}{\mu}\frac{\partial}{\partial r}\left\{\frac{Q_1\mu}{4\pi kh}\left[E_i\left(-\frac{r^2}{4\chi t}\right) + \sum\nolimits_{i=2}^{N}\frac{Q_i - Q_{i-1}}{Q_1}E_i\left(-\frac{r^2}{4\chi(t - t_{i-1})}\right)\right]\right\} \quad . \tag{7}$$

Mathematical simplification applied on equation (7) gives

[1] Both mathematical statement and an idea of using the formula (5) for pressure distribution belong to Dr. Fikri J. Kuchuk.

$$r \cdot u(r,t) = -\frac{\bar{c}Q_1}{2\pi h}\left(\exp\left(-\frac{r^2}{4\chi t}\right) + \sum_{i=2}^{N}\frac{Q_i - Q_{i-1}}{Q_1}\exp\left(-\frac{r^2}{4\chi\left(t - t_{i-1}\right)}\right)\right). \qquad (8)$$

Thus, we have a system describing the dynamic of characteristic functions obtained from the system (4)

$$\begin{cases} \dfrac{dr^2}{dt} = -\dfrac{\bar{c}Q_1}{\pi h}\left(\exp\left(-\dfrac{r^2}{4\chi t}\right) + \sum_{i=2}^{N}\dfrac{Q_i - Q_{i-1}}{Q_1}\exp\left(-\dfrac{r^2}{4\chi\left(t - t_{i-1}\right)}\right)\right), \\ r^2(0) = r_1^{\,2}; \end{cases} \qquad (9)$$

Numerical solution of the differential equation (9) creates characteristic families $r(t)$ for different r_1. Mathematically, the system (9) describes the dynamic of a control volume unit started from a point r_1 in the reservoir and appeared at the well sandface restricted by wellbore radius. The control volume unit changes its temperature during flowing through the reservoir with certain convection velocity; according to equation (4) the temperature perturbations are predicted by the following equation:

$$T\left(r(t,r_1),t\right) = f(r_1) + \varepsilon\left(P_i - p(r_t,t)\right) + (\varepsilon + \eta\Omega)\int_0^t \psi(r_\tau,\tau)d\tau \qquad (10)$$

where

$$\psi(r_\tau,\tau) = \frac{\partial p(r_\tau,\tau)}{\partial \tau} = -\frac{Q_1\mu}{4\pi kh}\left(\frac{e^{-\frac{r_\tau^{\,2}}{4\chi t}}}{\tau} + \sum_{i=2}^{N}\frac{Q_i - Q_{i-1}}{Q_1}\frac{e^{-\frac{r_\tau^{\,2}}{4\chi(\tau - t_{i-1})}}}{\left(\tau - t_{i-1}\right)}\right).$$

Summary. The temperature could be calculated by equation (10) for a given initial temperature profile $f(r)$ at the moment *t=0*. If we know temperature distribution at $t = t_{i-1}$ temperature prediction could be obtained for the moment $t_{i-1} < t < t_i$ using

$$T(r,t_i) = f(r_1) + \varepsilon\left(p(r_1,t_{i-1}) - p(r,t_i)\right) + (\varepsilon + \eta\Pi)\int_{t_{i-1}}^{t_i}\psi(r_\tau,\tau)d\tau \qquad (11)$$

A calculation algorithm has been designed on the basis on equation (11) for wellbore temperature prediction for a given production history.

Algorithm Testing. The temperature calculation results have been compared with Chekalyuk’s formula which is used for temperature calculation in case of constant (fixed) flow rate $Q_i = Q = const$ [7]:

$$\Delta P(t)\big|_{Q=const} = -\frac{Q\mu}{4\pi kh} E_i\left(-\frac{r_w^{\;2}}{4\chi t}\right) \tag{12}$$

$$\Delta T(t)\big|_{Q=const} = -\frac{\varepsilon Q\mu}{4\pi kh}\left[E_i\left(-\frac{r_w^{\;2}}{4\chi t}\right) - (1+\alpha)E_i\left(-\frac{r_w^{\;2}}{4\chi t} - a\right)\right] \tag{13}$$

here $\alpha = \dfrac{\eta\varphi\bar{c}}{\varepsilon}$ and $a = \dfrac{Q\varphi\mu\bar{c}}{4\pi kh}$.

Physical properties used for temperature calculation:

$P_i = 200\ atm$	$k = 1.0 \cdot 10^{-13}\ m^2$	$r_w = 0.1m;\ \phi = 0{,}2$	$\varepsilon = 0.04 \cdot 10^{-5}\ K/Pa$
$q = \dfrac{Q}{h} = 50\ \ m^2/day$	$\mu = 0.003\ Pa \cdot c$	$\eta = 0.014 \cdot 10^{-5}\ K/Pa$	$c_t = 10^{-9}\ 1/Pa$

Figures 1 and 2 shows satisfied matching of temperature data calculated from equations (10) and (13). A relative deviation does not exceed 0.5 %.

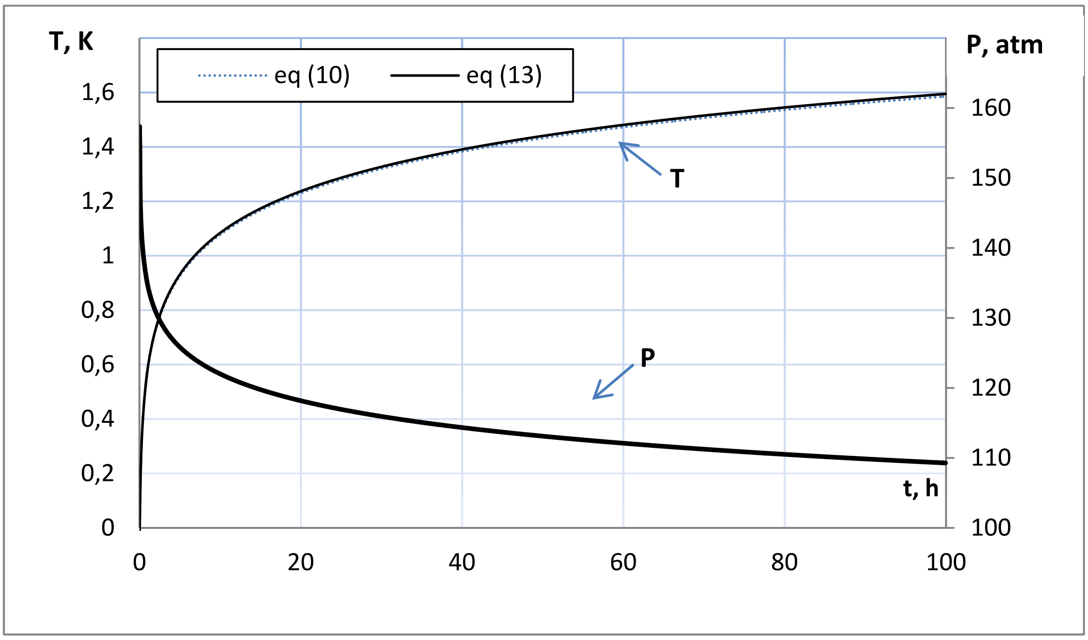

Figure 1. Wellbore pressure and temperature calculated from equations (11) and (13).

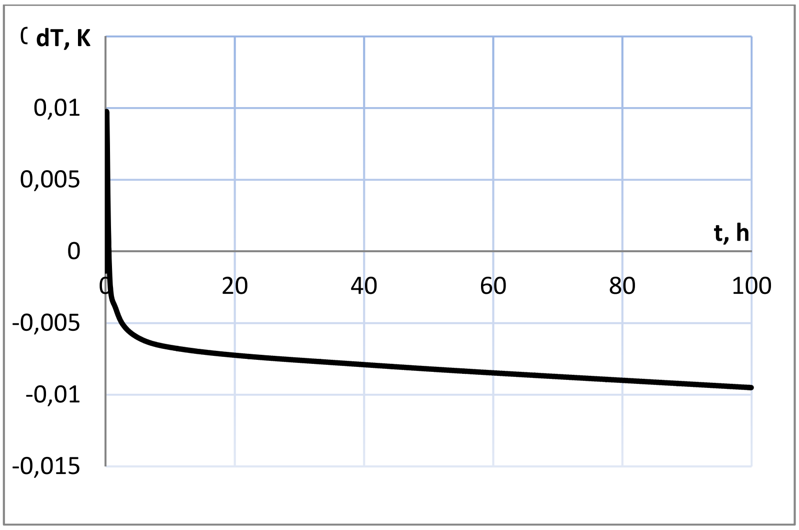

Figure 2. Temperature deviation derived from equations (11) and (13).

2. Mathematical temperature model with heat conduction

Mathematical statement. For this time, consider we have non-isothermal fluid flow through a porous media, but the mathematical temperature model is accounting for radial heat conduction additionally. Here we have the same assumptions described above in part 1 including adiabatic consideration. Heat transfer equation has almost the same view as in expression (1) except it has a heat conductivity term:

$$\frac{\partial T}{\partial t}+u(r,t)\frac{\partial T}{\partial r}=a\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)-\varepsilon u(r,t)\frac{\partial p(r,t)}{\partial t}+\eta\Pi\frac{\partial p(r,t)}{\partial t} \quad (14)$$

Initial temperature profile has a view:

$$T(r,0)=f(r) \quad (15)$$

Calculational Method. The problem (14, 15) has not a direct analytical solution. As described in [12] we can use a semi-analytical method OSATS (Operator Splitting and Time Stepping) [13]. The problem is divided on two parts according to this method: convection part and diffusion part. Let $T_{i-1}=T(r,t_{i-1})$ be the solution in (*i-1)* time slice. Consider Δt be a time step. The solution for the next time slice at the moment $t_i=t_{i-1}+\Delta t$ could be presented as following:

$$T_i=D_i(\Delta t)C_i(\Delta t)T_{i-1}, \quad (16)$$

Here operator $C_i(\Delta t)$ represents the solution of convective part of problem (14) which determines baro-thermal effect; $D_i(\Delta t)$ represents the solution of diffusion part defined as a heat conductivity part.

The solution of convective part of problem (14) is described by the equation (10). Let $\varphi(r,t_{i-1}) = C_i(\Delta t)T_{i-1}$ be a solution of convective part, by the same time it is an initial condition for a diffusion part representing the heat conductivity term. Thus, we have a one-dimension unsteady state problem for heat conduction:

$$\frac{\partial\varphi}{\partial t} - a\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0, \qquad t > t_{i-1} \tag{17}$$

With initial condition as a function:

$$\varphi(r,t_{i-1}) = C_i(\Delta t)T_{i-1}. \tag{18}$$

The solution for problem (17, 18) is known [14, 15]:

$$\varphi(r,t_i) = \frac{1}{2a\Delta t}\int_0^\infty \varphi(x,t_{i-1})e^{-\frac{r^2+x^2}{4a\Delta t}} I_0\left(\frac{rx}{2a\Delta t}\right)xdx \tag{19}$$

The expression (19) gives a temperature value $T_i = \varphi(r,t_i)$ on the current time slice t_i

3. Analysis of calculational results

Case 1. Here wellbore temperature predictions are presented for a given production history. Both cases are modeled: T1 – no heat conduction, T2 – temperature prediction accounting for radial heat conduction.

Physical properties used for temperature calculation:

$P_i = 200\ atm$	$k = 1.0\cdot10^{-13}\ m^2$	$r_w = 0.1m;\ \phi = 0{,}2$	$\varepsilon = 0.04\cdot10^{-5}\ K/Pa$
$a = 7.8\cdot10^{-7}\ m^2/\sec$	$\mu = 0.003\ Pa\cdot c$	$\eta = 0.014\cdot10^{-5}\ K/Pa$	$c_t = 10^{-9}\ 1/Pa$

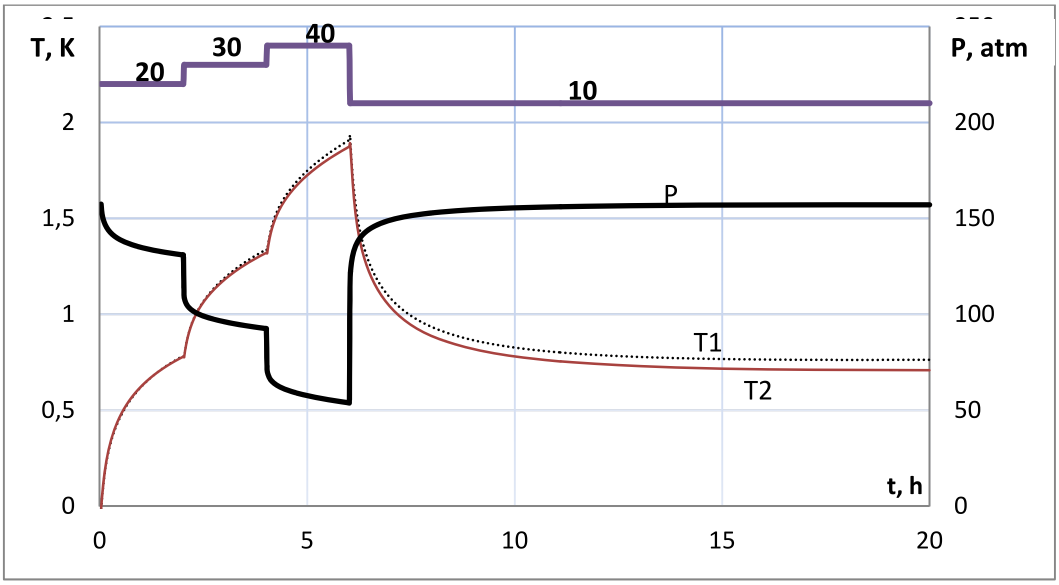

Figure 3. Wellbore pressure and temperature prediction:
T1 – no heat conduction, T2 – accounting for heat conduction.
Specific flow rate is indicated on figure by digits (m^2/day).

Figure 3 shows that the moments when flow rate is changed the wellbore pressure starts rapidly changing as well. It causes the temperature perturbations due to thermal expansion. Then Joule-Thomson effect prevails which is clearly noticed as a heating on temperature curves. The heat conduction could be obviously seen on the last cycle of the production history by temperature decreasing. Here the convective part is insufficient due to low flow rate and temperature deviation between T1 and T2 is becoming essential.

Case 2. Temperature prediction for shut-in period in production history.

Here let's consider we have a shut-in period in production history. Physical properties is the same as a previous case.

Let's look over the shut-in period. Figure 4 shows that at the moment when well had been shut-in the wellbore pressure increased very rapidly. Temperature simultaneously increased due to thermal compression (adiabatic effect). The intensity of pressure build-up was decreasing after a short time period; the adiabatic effect was insufficient by the same time. Then temperature started decreasing by the end of shut-in due to radial heat conduction.

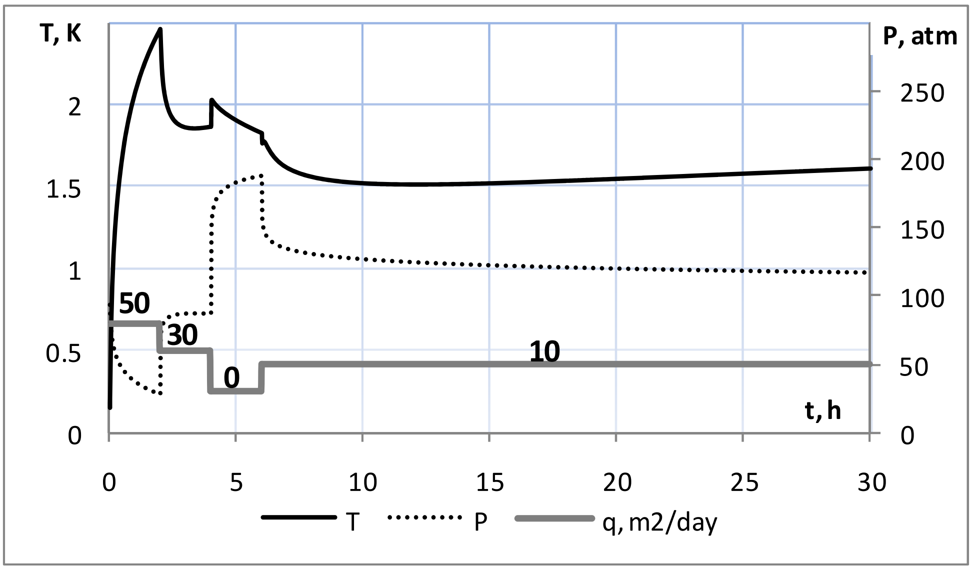

Figure 4. Wellbore pressure and temperature prediction: shut-in period in the production history. Specific flow rate is indicated on figure by digits (m^2/day).

Conclusions:

1. Heat transfer problem was solved for two cases: with/without heat conduction. Temperature was predicted for a give production history, for example, in case of Multi Rate Test.
2. Computation algorithm was designed for temperature calculation.
3. Computer software was developed on the basis of described computation algorithm for temperature prediction.
4. Algorithm and software were tested using analytical solution of temperature calculation in case of constant (fixed) flow rate without radial heat conduction.

Described mathematical model could be used for the following applications:

- Temperature prediction for a given production history and reservoir properties;
- Inverse problem solution: reservoir properties estimation on the basis of thermohydrodynamics data recorded downhole.

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ТЕМПЕРАТУРНОЕ ПОЛЕ В ПЛАСТЕ С УЧЕТОМ ТЕРМОДИНАМИЧЕСКИХ ЭФФЕКТОВ ПРИ РАБОТЕ СКВАЖИНЫ С ПЕРЕМЕННЫМ ДЕБИТОМ

Рамазанов А.Ш., Нагимов В.М., Ахметов Р.К.
ФГБОУ ВПО Башкирский государственный университет, г.Уфа
e-mail: ramaz@geotec.ru

Аннотация. Разработана математическая модель изменения температуры в нефтяном пласте для заданной истории дебита, включающая конвективный перенос тепла, радиальную теплопроводность, дроссельный и адиабатический эффекты. Разработаны алгоритм и компьютерная программа для расчета температурного поля по заданной истории изменения дебита жидкости из пласта. Проведено сравнение результатов расчетов с известными аналитическими решениями. Данную модель можно использовать для расчета изменения температуры на стенке скважины при известных истории изменения дебита и параметрах пласта, а так же при решении обратной задачи об определения параметров пласта по данным об изменении температуры в скважине.

Ключевые слова: температурное поле пласта, переменный дебит, аналитическая модель, расчет температуры, эффект Джоуля-Томсона, адиабатический эффект, баротермический эффект, теплопроводность, метод характеристик, обратная задача, формула Чекалюка

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Сведения об авторах

1. Рамазанов А. Ш., д.т.н., проф. кафедры геофизики, ФГБОУ ВПО БашГУ
A. Ramazanov, dr. tech. sci., prof. of the geophysical department, FSBEI BSU

2. Нагимов В. М., соискатель кафедры геофизики, ФГБОУ ВПО БашГУ
V. Nagimov, applicant of the geophysical department, FSBEI BSU

3. Ахметов Р.К., магистрант кафедры геофизики, ФГБОУ ВПО БашГУ
R. Akhmetov, master of science of the geophysical department, FSBEI BSU

e-mail: ramaz@geotec.ru