THE EVALUATION OF POSSIBILITIES OF AUXILIARY WELLBORES TESTING USING TEMPERATURE CHANGES IN THE WELL

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In this paper the question of utilizing borehole temperature measurements potentialities to determine inflow intervals of the auxiliary wellbores without getting into auxiliary wellbore is studied using mathematical calculation. Convection heat transfer in auxiliary wellbore and thermodynamic effects (barothermal and throttling or Joule - Thomson effect) in formation are taken into account.

Keywords: *well, auxiliary wellbore, temperature, throttling effect, heat exchange, inflow intervals*

During the last few years in the oil field development drilling of extra auxiliary wellbores from the main wellbore became widely spread to increase the productivity of the well [1,2]. In that case the problem of auxiliary wellbores testing appears on the stages of well development and well operation, the following problems are solved within it:

- locating of the inflow intervals to the auxiliary wellbores;
- determination of the flow rate and the nature of inflow (oil, water, gas).

In this paper the question of utilizing borehole temperature measurements potentialities to determine operational intervals of the auxiliary wellbores without getting into auxiliary wellbore is studied using mathematical calculation. The temperature distribution in the well can be obtained by the conventional temperature survey using temperature sensor moving along the borehole. One can use multisensor technology when the autonomous temperature gauges are located on the different depths below and above the point of junction of the auxiliary wellbore with the well. The similar data can be obtained using fiber-optic temperature measurements or disturbed temperature sensor (DTS) technology.

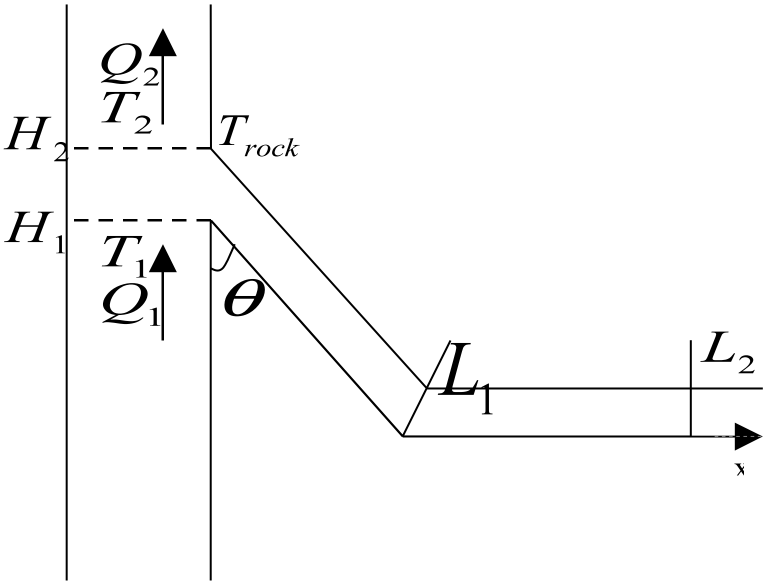

Figure 1. *Schematic sketch of the borehole and auxiliary wellbore*

T_1 and T_2 are the temperatures at depth H_1 and H_2 (below and above of the point of junction of auxiliary wellbore with borehole), respectively. Q_1 and Q_2 are the flow rates below and above of the point of junction of auxiliary wellbore with borehole, respectively. T_{rock} is the temperature of surrounding rocks; θ is a tilt angle of a part of the auxiliary wellbore.

If we know value of temperatures T_1 and T_2 (Fig. 1), flow rates Q_1 and Q_2 (Fig.1), volumetric heat capacity C_1 on depth H_1 (below the point of junction) and H_2 (above the point of junction) (Fig. 1) from the results of borehole analysis then temperature of the fluid coming from the auxiliary wellbore can be found using following expression:

$$T=\frac{T_2\left[C_{aw}\left(Q_2-Q_1\right)+C_1 Q_1\right]-C_1 Q_1 T_1}{C_{aw}\left(Q_2-Q_1\right)},$$

where C_{aw} is a volumetric heat capacity of fluid coming from the auxiliary wellbore.

Obviously, investigation of steady temperature field in the well will not allowed us to locate operational intervals in the auxiliary wellbore. Therefore for further studies the pattern of temperature changes of fluid coming from the auxiliary wellbore for the transient behaviour *T(t)* is used, e.g., just after the well have started operating.

Let us examine few problems on temperature changes of the fluid coming from the auxiliary wellbore.

Problem 1. *It is necessary to determine value of temperature signal on the way out from the auxiliary wellbore as a function of time and also depending on the borehole configuration, value of temperature signal in the inflow interval, velocity of fluid in the auxiliary wellbore, thermal conductivity between auxiliary wellbore and surrounding rocks, tilt angle θ (Fig. 1) of a part of the auxiliary wellbore.*

The signal propagates from the boundary $x = L_2$ along the horizontal part and then from $x = L_1$ along the inclined part of the auxiliary wellbore with radius R at a slope θ (Fig.1).

Model solution can be obtained by solving following system:

$$\begin{cases} \frac{\partial T}{\partial t} - \upsilon \frac{\partial T}{\partial x} = \beta (T_{rock} - T) \\ T|_{t=0} = T_{rock} = \begin{cases} T_0 + \Gamma x \cdot \cos\theta, x \le L_1 \\ T_0 + \Gamma L_1 \cdot \cos\theta, x > L_1 \end{cases}, \\ T|_{x=L_1} = R(T) \\ T|_{x=L_2} = \Delta T_0 \end{cases} \quad (1)$$

where T is an average temperature along the auxiliary wellbore cross section, T_{rock} is a temperature of surrounding rocks, $\beta = \frac{2\alpha}{C_1 \rho_1 R}$, where α is a coefficient accounting for heat exchange between the auxiliary wellbore and surrounding rocks, υ is fluid velocity in the auxiliary wellbore, Γ is a geothermal gradient.

Substituting $U(x,t) = T(x,t) - \begin{cases} T_0 + \Gamma x \cdot \cos\theta, \ x \le L_1 \\ T_0 + \Gamma L_1 \cdot \cos\theta, \ x > L_1 \end{cases}$ in system (1) we get:

$$\begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} - \upsilon \Gamma \cos\theta = -\beta U \\ U|_{t=0} = T|_{t=0} - T_{rock} = 0 \\ U|_{x=L_1} = \psi(t) \\ x \le L_1 \\ U|_{t=0} = T|_{t=0} - T_{rock} = 0 \\ U|_{x=L_1} = \Delta T_0 \\ L_1 < x < L_2 \end{cases} \quad (1.1)$$

From (1.1) we can mark out two problems:

$$x \le L_1 : \begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} - \upsilon \Gamma \cos\theta = -\beta U \\ U|_{t=0} = 0 \\ U|_{x=L_2} = \psi(t) \end{cases} \quad (1.1')$$

$$L_1 < x < L_2 : \begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} = -\beta U \\ U|_{t=0} = 0 \\ U|_{x=L_2} = \Delta T_0 \end{cases} \quad (1.2')$$

Firstly, we solve (1.2') using Laplace transform in order to obtain the function *ψ(t)* we are interested in to resolve system (1.1').

$$t \to S, U(x,t) \to F(S), F(S) = \int_0^\infty U(x,t) \cdot e^{-St} dt$$

After the transformation:

$$\begin{cases} SF - \upsilon \frac{\partial F}{\partial x} = -\beta F \\ F|_{t=0} = 0 \\ F|_{x=L_2} = \frac{\Delta T_0}{S} \\ L_1 < x < L_2 \end{cases} \quad (1.3)$$

From the system (1.3) we get:

$$F = C \exp\left(\frac{S+\beta}{\upsilon} x\right),$$

where $C = \frac{\Delta T_0}{S} \exp(-\frac{S+\beta}{\upsilon} L_2)$ from the boundary conditions.

Then we resolve *F*:

$$F = \frac{\Delta T_0}{S} \exp\left(-\frac{S+\beta}{\upsilon}(L_2 - x)\right)$$

Further, applying inverse Laplace transform one can get:

$$U(x,t) = \Delta T_0 \exp(-\beta \frac{L_2 - x}{\upsilon}) \gamma (t > \frac{L_2 - x}{\upsilon}) \quad (1.4)$$

$$\psi(t) = U(L_1, t) : \; U(L_1, t) = \Delta T_0 \exp(-\beta \frac{L_2 - L_1}{\upsilon}) \gamma (t > \frac{L_2 - L_1}{\upsilon}) \quad (1.5)$$

To find the solution to the system (1.1') we use Laplace transform:

$$\begin{cases} SF-\upsilon\dfrac{\partial F}{\partial x}-\dfrac{\Gamma\upsilon\cos\theta}{S}=-\beta F \\ F|_{t=0}=0 \\ F|_{x=L_1}=\dfrac{\Delta T_0}{S}e^{-t_2(S+\beta)} \\ x<L_1 \end{cases} \tag{1.6}$$

$$F=\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}+Ae^{\frac{S+\beta}{\upsilon}x}$$

$$A=F|_{x=L_1}e^{-\frac{S+\beta}{\upsilon}L_1}-\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}e^{-\frac{S+\beta}{\upsilon}L_1}$$

$$F=\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}+\frac{\Delta T_0}{S}e^{-\frac{S+\beta}{\upsilon}(t_2+t_1-\frac{x}{\upsilon})}-\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}e^{-\frac{S+\beta}{\upsilon}(L_1-x)} \tag{1.7}$$

Utilizing inverse Laplace transform we obtain *U(x,t)*:

$$\begin{aligned} U(x,t)=&\Gamma\upsilon\cos\theta\frac{1-e^{-\beta\cdot t}}{\beta}+\Delta T_0 e^{-\beta(t_2+t_1-\frac{x}{\upsilon})}\gamma\left(t>t_2+t_1-\frac{x}{\upsilon}\right)+ \\ &+\Gamma\upsilon\cos\theta\, 1-\frac{e^{-\beta(t-t_1-\frac{x}{\upsilon})}}{\beta}e^{-\beta(t_1-\frac{x}{\upsilon})})\gamma\left(t>t_1-\frac{x}{\upsilon}\right) \end{aligned} \tag{1.8}$$

From (1.8) we can get value of the temperature anomaly at the point where the temperature signal goes out from the inclined-horizontal auxiliary wellbore:

$$U(0,t)=\Gamma\upsilon\cos\theta\frac{1-e^{-\beta\cdot t}}{\beta}+\Delta T_0 e^{-\beta(t_2+t_1)}\gamma\left(t>t_2+t_1\right)+\Gamma\upsilon\cos\theta\frac{1-e^{-\beta(t-t_1)}}{\beta}e^{-\beta\cdot t_1}\gamma\left(t>t_1\right)$$

$$U(0,t)=\begin{cases} \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t}}{\beta},\ t<t_1 \\ \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t_1}}{\beta},\ t_2+t_1>t\geq t_1 \\ \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t_1}}{\beta}+\Delta T_0 e^{-\beta(t_2+t_1)},\ t\geq t_2+t_1 \end{cases} \tag{1.9}$$

Without regard for the heat exchange the temperature anomaly on the way out from the auxiliary wellbore can be written as

$$U(0,t)=\begin{cases} \Gamma\upsilon\cos\theta\cdot t,\ t<t_1 \\ \Gamma\upsilon\cos\theta\cdot t,\ t_2+t_1>t\geq t_1 \\ \Gamma\upsilon\cos\theta\cdot t_1+\Delta T_{0,} t\geq t_2+t_1 \end{cases} \tag{1.10}$$

The calculations are made using obtained expressions (1.9) and (1.10) for the following values of parameters: *Γ=0.014 K/m, Q=10 m³/day, α=6 Wt/m²/K, C=2000 J/kg/К, ρ=800 kg/m³, L₂=100 m, L₁=50 m, R=0.1 m, ΔT=0,15 K***.**

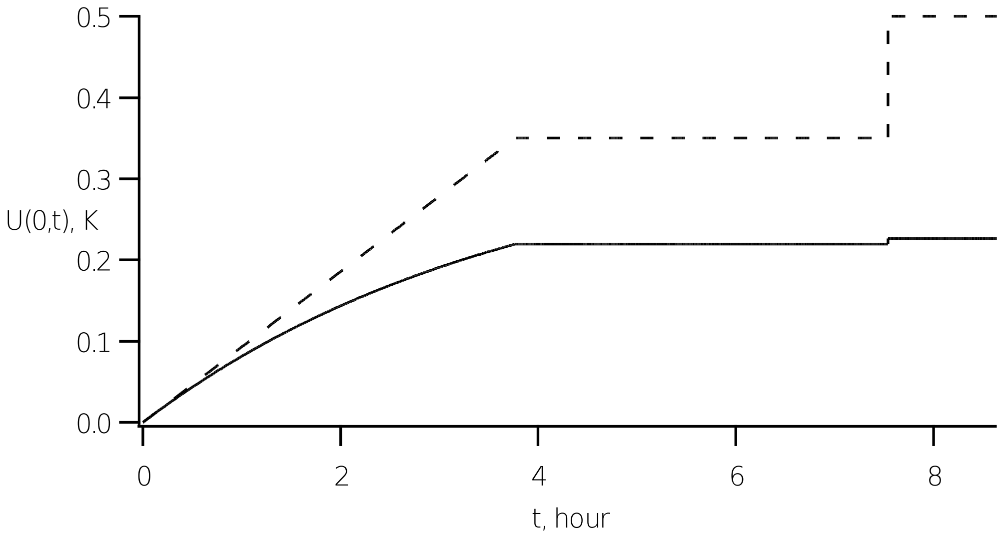

Figure 2. Value of the temperature anomaly
on the way out from the auxiliary wellbore as a function of time.

Formulas (1.9) and (1.10) allow to estimate decreasing of temperature signal, caused by fluid inflow from formation to auxiliary wellbores at the further fluid flow in auxiliary wellbore due to the heat transfer with rock.

Problem 2. *It is necessary to find the dependence of transient temperature on the way out from the auxiliary wellbore from the transient temperature in the interval of inflow to the auxiliary wellbore.*

In this case the temperature signal also propagates from the boundary $x = L_2$ (temperature here is a function of time, we will consider arbitrary function) along the horizontal part and then from $x = L_1$ along the inclined at an angle θ (Fig. 1) part of the auxiliary wellbore with radius R (Fig. 1).

$$\begin{cases} \frac{\partial T}{\partial t} - \upsilon \frac{\partial T}{\partial x} = \beta (T_{nop} - T) \\ T|_0 = T_{nop} = \begin{cases} T_0 + \Gamma x \cdot \cos\theta ,\ x \le L_1 \\ T_0 + \Gamma L_1 \cdot \cos\theta ,\ x > L_1 \end{cases} \\ T|_{x=L_2} = T_0 + \Gamma L_1 \cdot \cos\theta + P(t) \end{cases} \quad (2)$$

In fact, solved problem one, where the temperature on the boundary $x = L_2$ is a constant value ΔT_0, is the special case of the problem two.

Substituting $U(x,t) = T(x,t) - \begin{cases} T_0 + \Gamma x \cdot \cos\theta, & x \le L_1 \\ T_0 + \Gamma L_1 \cdot \cos\theta, & x > L_1 \end{cases}$ in system (2) we get:

$$\begin{cases} \begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} - \upsilon \Gamma \cos\theta = -\beta U \\ U|_{t=0} = T|_{t=0} - T_{rock} = 0 \\ U|_{x=L_1} = \psi(t) \\ x \le L_1 \end{cases} \\ \begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} = -\beta U \\ U|_{t=0} = T|_{t=0} - T_{rock} = 0 \\ U|_{x=L_2} = \varphi(t) \\ L_1 < x < L_2 \end{cases} \end{cases} \quad (2.1)$$

From (2.1) we can mark out two problems:

$$x \le L_1 : \begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} - \upsilon \Gamma \cos\theta = -\beta U \\ U|_{t=0} = 0 \\ U|_{x=L_1} = \psi(t) \end{cases} \quad (2.1')$$

$$L_1 < x < L_2 : \begin{cases} \frac{\partial U}{\partial t} - \upsilon \frac{\partial U}{\partial x} = -\beta U \\ U|_{t=0} = 0 \\ U|_{x=L_2} = \varphi(t) \end{cases} \quad (2.2')$$

Similarly to the system (1.2') we solve system (2.2') and find:

$$U(x,t) = \varphi\left(t - \frac{L_2 - x}{\upsilon}\right) \exp\left(-\beta \frac{L_2 - x}{\upsilon}\right) \gamma\left(t > \frac{L_2 - x}{\upsilon}\right)$$

$$\psi(t) = U(L_1, t) = \varphi\left(t - \frac{L_2 - L_1}{\upsilon}\right) \exp\left(-\beta \frac{L_2 - L_1}{\upsilon}\right) \gamma\left(t > \frac{L_2 - L_1}{\upsilon}\right) \quad (2.3)$$

$$\frac{L_2 - L_1}{\upsilon} = t_2 \Rightarrow \psi(t) = \varphi(t - t_2) \exp(-\beta t_2) \gamma(t > t_2)$$

Then we can solve (2.1') utilizing Laplace transform and get:

$$F=\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}+Ae^{\frac{S+\beta}{\upsilon}x}$$

$$A=F|_{x=L_1}e^{-\frac{S+\beta}{\upsilon}L_1}-\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}e^{-\frac{S+\beta}{\upsilon}L_1}$$

$$F=\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}+\varphi(t)e^{-\frac{S+\beta}{\upsilon}(L_1-x)}-\frac{\Gamma\upsilon\cos\theta}{S(S+\beta)}e^{-\frac{S+\beta}{\upsilon}(L_1-x)}\text{, then}$$

$$U(x,t)=\Gamma\upsilon\cos\theta\frac{1-e^{-\beta\cdot t}}{\beta}+\psi\left(t-\frac{L_1-x}{\upsilon}\right)e^{-\beta\left(\frac{L_1-x}{\upsilon}\right)}\gamma\left(t>\frac{L_1-x}{\upsilon}\right)+$$
$$+\Gamma\upsilon\cos\theta\frac{1-e^{-\beta\left(t-\frac{L_1-x}{\upsilon}\right)}}{\beta}e^{-\beta\left(\frac{L_1-x}{\upsilon}\right)}\gamma\left(t>\frac{L_1-x}{\upsilon}\right)$$

At the point on the way out of the fluid from the auxiliary wellbore (*x = 0*) we have:

$$U(0,t)=\Gamma\upsilon\cos\theta\frac{1-e^{-\beta\cdot t}}{\beta}+\psi(t-t_1)e^{-\beta\left(\frac{L_1-x}{\upsilon}\right)}\gamma(t>t_1)+ \\ +\Gamma\upsilon\cos\theta\frac{1-e^{-\beta(t-t_1)}}{\beta}e^{-\beta t_1}\gamma(t>t_1) \tag{2.4}$$

From (2.4) we can make a system:

$$U(0,t)=\begin{cases}\Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t}}{\beta}, t<t_1\\ \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t_1}}{\beta}, t_2+t_1>t\geq t_1\\ \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t_1}}{\beta}+\varphi(t-t_1-t_2)e^{-\beta(t_1+t_2)}, t\geq t_2+t_1\end{cases} \tag{2.5}$$

Without regard for the heat exchange the temperature anomaly on the way out from the auxiliary wellbore can be written as

$$U(0,t)=\begin{cases}\Gamma\upsilon\cos\theta\cdot t, t<t_1\\ \Gamma\upsilon\cos\theta\cdot t_{1,}t_2+t_1>t\geq t_1\\ \Gamma\upsilon\cos\theta\cdot t_1+\varphi(t-t_1-t_2), t\geq t_2+t_1\end{cases} \tag{2.6}$$

In our model we assume [3]:

$$\varphi(t)=\frac{\varepsilon Q\mu}{4\pi\kappa L}\ln\left(1+2\frac{cQt}{\pi L r_c^2}\right). \tag{2.7}$$

With regard for (2.7), expressions (2.5) and (2.6) can be rewritten as

$$U(0,t)=\begin{cases} \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t}}{\beta}, t<t_1 \\ \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t_1}}{\beta}, t_2+t_1>t\geq t_1 \\ \Gamma\upsilon\cos\theta\dfrac{1-e^{-\beta\cdot t_1}}{\beta}+\dfrac{\varepsilon Q\mu}{4\pi\kappa L}\ln\left(1+2\dfrac{cQ(t-t_1-t_2)}{\pi L r_c^2}\right)e^{-\beta(t_1+t_2)}, t\geq t_2+t_1 \end{cases} \quad (2.8)$$

$$U(0,t)=\begin{cases} \Gamma\upsilon\cos\theta\cdot t, t<t_1 \\ \Gamma\upsilon\cos\theta\cdot t_1, t_2+t_1>t\geq t_1 \\ \Gamma\upsilon\cos\theta\cdot t_1+\dfrac{\varepsilon Q\mu}{4\pi\kappa L}\ln\left(1+2\dfrac{cQ(t-t_1-t_2)}{\pi L r_c^2}\right)e^{-\beta(t_1+t_2)}, t\geq t_2+t_1 \end{cases} \quad (2.9)$$

The calculations are made using obtained expressions (2.8) and (2.9) for the following values of parameters: *Γ=0.014 K/m, Q=10* m^3*/day, α=6 Wt/m²/K, C=2000 J/kg/K, ρ=800 kg/*m^3*, L2=100 m, L1=50 m,* r_c*=0.1 m, c=0.8, μ =10 kPs, κ=0.1 D, ε =0.04K/atm.*

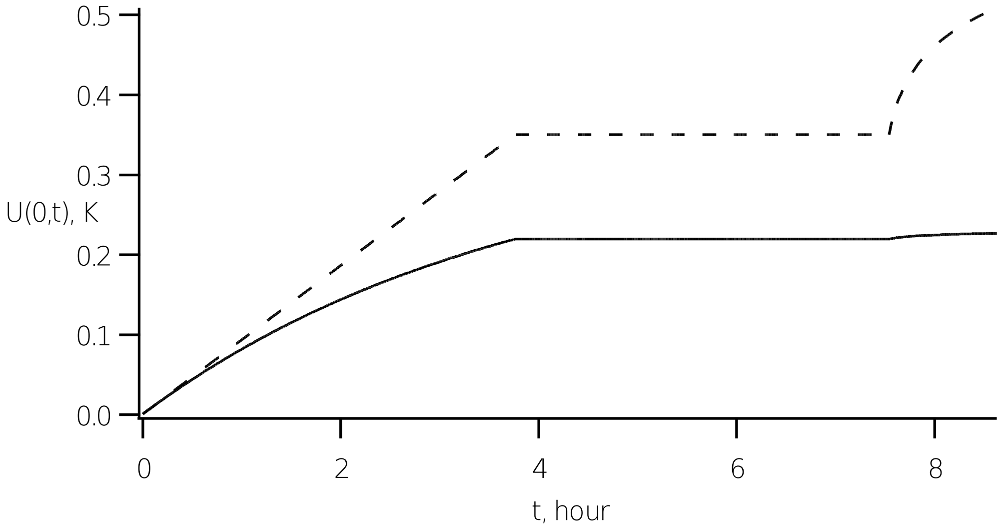

Figure 3. Value of the temperature anomaly
on the way out from the auxiliary wellbore as a function of time

On the fig. 4 it is seen, that the temperature change on the outflow from auxiliary wellbore occurs by phases, which are characterized by the time of thermal front propagation from $x=L_1$ and $x=L_2$. From the comparison of formulas (2.8) and (2.9), it is also seen, that character of temperature change in time, which is formed in the inflow interval in auxiliary wellbore, preserves on the way out from the auxiliary wellbore too. Value of temperature change is decreasing due to the heat transfer with the surroundings of auxiliary well bore, as in the case of constant temperature anomaly (compare formulas (1.9) and (2.8)). Decreasing of temperature signal can be significant, as on the fig. 3, it depends on the coefficient β value and time of temperature front movement in auxiliary wellbore.

CONCLUSION

In this work we built mathematical models for heat-and-mass transfer in the auxiliary wellbore, which allows us to:

- investigate the nature of thermal anomaly changes on the way out from the auxiliary wellbore from the different parameters of model;
- solve inverse problems on defining operational intervals of the auxiliary wellbore.

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