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SPATIAL PARAMETRICAL VIBRATIONS OF A PIPELINE AFFECTED BY VARIABLE INTERNAL PRESSURE.
RESEARCH ON THE EFFECTS OF CHARACTERISTICS OF INTERNAL PRESSURE IN A LIQUID, ARCHIMEDES FORCE, CORIOLIS INERTIA FORCES AND DRAG FORCES ON FLEXURAL AND TORSIONAL VIBRATIONAL MOVEMENTS OF A PIPELINE. P. II

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Abstract. In we put forward an approximate mathematical model of flexural-rotary vibrational movements in a twin pipeline bent by gravity. The model was developed in an assumption of low pillar elasticity and pipe deformations associated with the pipe leave out of the flexure plane and takes account for the Archimedes buoyancy force, Coriolis inertia forces and drag forces proportional to the first-rate velocity. We gave formulas describing natural frequencies of pipeline flexural-rotary vibrations and determined the effects of the Archimedes force, Coriolis inertia forces, drag forces and also geometrical and physical mechanical parameters of the pipe on its natural vibrations.

Keywords: spatial parametric vibrations, pipeline, alternating internal pressure, static pressure, initial phase, amplitude, circular frequency of pressure alterations.

The present paper being a continuation of the work begun in [1] is devoted to solving the problem on spatial flexural-rotary vibrations of a pipeline under the action of alternating internal pressure. Special attention is paid to research on the effects of values of static and dynamic components, circular frequency and initial phase of pressure in a liquid, Archimedes buoyancy force, Coriolis inertia forces and drag forces on flexural and rotary vibrational movements of the pipeline.

A calculation scheme for flexural and rotary vibrations produced in a pipeline is given in Figure 1. The left side of Figure 1 shows a pipe portion of length $dx$ and mass $dm = \frac{m}{L} dx$, and the right side of the same figure demonstrates accelerations and forces acting on the selected pipe portion. The length of the pipe is $L$, the thickness of its wall is $h$, and the total mass of the homogeneous pipe and liquid is $m$. 

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The load \( q_n \) distributed along the pipeline is expressed by the formula [2]:

\[
q_n = -\frac{m}{L} \frac{\partial^2 w}{\partial t^2} + P_i \frac{\partial^2 w}{\partial x^2}, \quad F_i = \pi R_i^2, \quad P_i = P_0 + P_a \sin(\Omega t + \varphi)
\]  

(1)

where \( w \) is the deflection of the pipe portion, \( \Omega, \varphi, P_0 \) and \( P_a \) are the circular frequency, the initial phase, the value of the static component value and the amplitude of the dynamic component of the alternating internal pressure \( P_i \) in the pipe, \( R_i, F_i \) are the internal radius and the area of the pipe flow section, \( t \) is the time.

The equation of the conditional equilibrium in the pipe as a sum of moments of all the applied forces and inertia forces about the axis \( Dx \) has the form

\[
-\int (dmg - dA) w \sin \theta - \int wd\Phi_z - \int wd\Phi_k - M_e - \mu \frac{d\theta}{dt} \int w^2 dx = 0,
\]

(2)

where \( \theta \) is the angle of rotation in the pipe as a rigid body about the axis \( Dx \), 

\( dA = \rho_{cs} \pi (R_i + h)^2 g dx \) is the Archimedes buoyancy force, 

\( d\Phi_z = dmw \frac{d^2 \theta}{dt^2} \),

\( d\Phi_k = 2dm \frac{d\theta}{dt} \frac{\partial w}{\partial t} \) are the tangential and Coriolis inertia forces of the pipe portion, \( \rho_{cs} \) is the density of the continuous medium around the pipe, \( g \) is the gravitational acceleration, \( M_e = c \cdot \theta \) is the total moment of the elastic forces in the pillars, \( c \) is the elasticity coefficient of the pillars, \( \mu \) is the drag coefficient depending on the continuous medium viscosity and the immersed body shape [3].

The differential equation of the pipe’s flexural vibrations in its own plane is as follows
\[ \frac{\partial^2 w}{\partial t^2} = -\frac{EJL}{m} \frac{\partial^4 w}{\partial x^4} + \frac{(T - PF_L)L}{m} \frac{\partial^2 w}{\partial x^2} + g_1 \cos \theta + \omega^2 \left( \frac{d\theta}{dt} \right)^2 - \frac{\mu L}{m} \frac{\partial w}{\partial t}, \]

where \( T = \frac{EF}{2L} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 \, dx \) is the longitudinal tension force, \( E, F = 2\pi R_h \) and \( J \approx \pi R^3_h \) is the Young’s modulus of the material, the cross sectional area and its axial moment of inertia in the pipeline, \( g_1 = g - \frac{A}{m}, A = \rho \omega \pi (R_i + h)^2 Lg \) is the Archimedes force expelling the whole pipe.

The function of the pipe’s deflection that satisfies the boundary conditions

\[ w(0,t) = w(L,t) = \frac{d^2 w}{dx^2}(0,t) = \frac{d^2 w}{dx^2}(L,t) = 0, \]

is taken in the form

\[ w(x,t) = (W_0 + w_0(t)) \sin \frac{\pi x}{L}, \]

where \( W_0 \) and \( w_0(t) \) are the amplitudes of the deflection static and dynamic components.

Inserting the functions (1) and (4) into the equations (2) and (3) and applying the Bubnov-Galerkin method [4] to the latter one, we obtain the following set of differential equations

\[ \frac{1}{2} \frac{d^2 \theta}{dt^2} \left( W_0 + w_0(t) \right)^2 + \frac{\mu}{m} \frac{d\theta}{dt} + \frac{c}{m} \theta + \left( W_0 + w_0(t) \right) \left( \frac{2g_1}{\pi} \sin \theta + \frac{d\theta}{dt} \frac{dw_0}{dt} \right) = 0, \]

\[ \frac{d^2 w_0}{dt^2} + \frac{\mu L}{m} \frac{dw_0}{dt} + \frac{\pi^4 EJ}{mL^3} \left( W_0 + w_0(t) \right) = \frac{4g_1}{\pi} \cos \theta + \left( W_0 + w_0(t) \right) \left( \frac{d\theta}{dt} \right)^2 - \frac{\pi^2 EF}{4L^2} \left( W_0 + w_0(t) \right)^2 - F_1 \left( P_0 + P_a \sin(\Omega t + \varphi) \right) \frac{\pi^2}{mL} \left( W_0 + w_0(t) \right). \]

The work [1] gives also an algebraic equation to determine the static component \( W_0 \) of the deflection in the pipe

\[ \frac{\pi^4 EJ}{4L^2} W_0^3 + \pi^2 \left( \frac{\pi^2 EJ}{L^2} - F_1 P_0 \right) W_0 - \frac{4g_1 mL}{\pi} = 0, \]

that makes it possible to find an approximate critical magnitude for the static pressure

\[ P_0 = \frac{\pi^2 EJ}{F_1 L^2}. \]
The set of equations (5) is solved under the initial conditions

\[ t=0, \quad \theta=\theta_0, \quad \frac{d\theta}{dt} = \omega_0, \quad w_0 = 0, \quad \frac{dw_0}{dt} = 0. \]  

Here, \( \theta_0, \omega_0 \) are the initial angle of rotation and the angular velocity of the pipe.

Let us consider the effect of the static component value and the dynamic component amplitude, the circular frequency and the initial pressure phase in liquid, the Archimedes buoyancy force, the Coriolis inertia forces and the drag forces acting on flexural and rotary vibration movements of the pipeline.

Here, as in the work [1], the numerical solution to the Cauchy problem (5), (8) was determined by the Runge-Kutta method. The calculation results for the following main parameter magnitudes: \( m = 6,141\cdot10^3 \) kg, \( L = 25 \) m, \( c = 0 \), \( g = 9,8 \) m/s\(^2\), \( R_0 = 0,259 \) m, \( h = 0,006 \) m, \( E = 2,0\cdot10^{11} \) Pa, \( \theta_0 = 0,3 \) rad \( \omega_0 = 0 \) rad/s are given in the form of diagrams. Figures 2-33 give the diagrams of time-dependencies of the angle of rotation \( \theta \) and the dynamic deflection \( w_0(t) \) in the middle point of the pipe span, respectively. In the diagrams, the solid lines depict the calculation results with account for drag forces whereas the dashed lines show the results with no account for these forces. The calculations were made for two sets of values for the drag coefficient \( \mu \) and the continuum density \( \rho_{os} \): \( \mu =24 \) Pa\(\cdot\)s and \( \rho_{os} =800 \) kg/m\(^3\) (aqueous medium), \( \mu =0,021 \) Pa\(\cdot\)s and \( \rho_{os} =1,25 \) kg/m\(^3\) (air medium). The diagrams given in Figures 2-17 and 18-33 illustrate the calculation results for the two above noted sets of values for the drag coefficient and the continuum density, respectively.

The effect of the value \( P_0 \) for the internal pressure static component in the pipeline on its flexural and rotary vibration movements is demonstrated in Figures 2-7, 18-23 and 8-12, 15-17, 24-29, 32, 33. The calculations were made for two values of the pressure static component: \( P_0=1,0 \) MPa (Figures 2-7, 18-23) and \( P_0=5,08 \) MPa (Figures 8-17, 24-33). In this case the value \( P_a \) for the pressure wave dynamic component had two magnitudes: \( P_a= 0,01 \) MPa, \( P_a =0,05 \) MPa, and the circular frequency \( \Omega \) and the initial phase \( \varphi \) had three magnitudes: \( \Omega =2,8 \) rad/s, \( \Omega =6,8 \) rad/s, \( \varphi =0 \) rad, \( \varphi =\pi/2 \) rad, \( \varphi =\pi \) rad. By comparing the corresponding diagrams in these figures, we can see that an increase in the amplitude of flexural vibrations and a decrease in the frequency of rotary vibrations occur simultaneously with the growing internal pressure static component at the similar parameter magnitudes. Besides, it can be noted that the greatest changes in the pipe flexural and rotary vibration amplitudes at the same static pressure \( P_0 \) are due to changing magnitudes of the circular frequency \( \Omega \). As seen from all these diagrams, if the pipe vibrations occur in aqueous medium when \( \mu =24 \) Pa\(\cdot\)s and \( \rho_{os} =800 \) kg/m\(^3\) the amplitudes of flexural and rotary vibrations with and without account for drag differ to a large extent (Figures 2-17). If the pipe produces vibrations in the air medium when \( \mu \)
=0.021 Pa·s and \( \rho_{os} = 1.25 \text{ kg/m}^3 \), we can see a coincidence of calculation results with and without account for drag forces (Figures 18-33). In the latter case we can also observe a pronounced increase in the frequencies of both rotary and flexural vibrations in the pipe.

In order to conclude about the effect exerted by changing amplitude \( P_a \) of the internal pressure dynamic component in the pipe on the latter’s flexural and rotary vibrations, we should compare the corresponding diagrams, e.g., in Figures 9 and 12 (aqueous medium, \( P_0 = 5.08 \text{ MPa}, \Omega = 6.8 \text{ rad/s}, \varphi = 0 \text{ rad} \), 26 and 29 (air medium, \( P_0 = 5.08 \text{ MPa}, \Omega = 10.8 \text{ rad/s}, \varphi = 0 \text{ rad} \)). In Figures 9 and 26, the diagrams are constructed for \( P_a = 0.01 \text{ MPa} \), and in Figures 12 and 29 for \( P_a = 0.05 \text{ MPa} \). As can be seen, if the pipe moves in aqueous medium with growing amplitude \( P_a \) of the internal pressure dynamic component, it is accompanied by a considerable increase in the amplitude of flexural vibrations and a less pronounced increase in the amplitude of rotary vibrations. If the pipe moves in air medium, an increase in the amplitude \( P_a \) of the internal pressure dynamic component leads to an increase in the amplitudes of both flexural and rotary vibrations. In the latter case we can also note a decrease in the frequencies of rotary and flexural vibrations.

For \( P_0 = 5.08 \text{ MPa} \) and \( P_a = 0.05 \text{ MPa} \), similar diagrams are constructed in Figures 13, 14 (\( \mu = 24 \text{ Pa·s}, \rho_{os} = 800 \text{ kg/m}^3, \Omega = 6.8 \text{ rad/s} \)) and 30, 31 (\( \mu = 0.021 \text{ Pa·s}, \rho_{os} = 1.25 \text{ kg/m}^3, \Omega = 10.8 \text{ rad/s} \)) with no account for the Archimedes buoyancy force (Figures 13 and 30) and the Coriolis inertia forces (Figures 14 and 31). If we compare the corresponding diagrams in Figures 12 and 13, 14, and also 29, 30 and 31, we can state the following.

If no account is taken for the Archimedes buoyancy force, a considerable increase in the frequencies and a decrease in the amplitudes occur simultaneously when the pipe moves in aqueous medium. In this case we can also note a considerable increase in the frequency of flexural beatings. In the case when the pipe moves in air medium, its frequencies and amplitudes of rotary and flexural vibrations change inconsiderably.

If no account is taken for the Coriolis inertia forces, both frequencies and amplitudes of rotary and flexural vibrations increase inconsiderably when the pipe moves in aqueous medium. In this case flexural vibrations with the highest amplitude continue over a shorter period of time. There is also a slight increase in the frequency of flexural beatings. In the case that the pipe moves in air medium, its frequencies and amplitudes of rotary and flexural beatings decrease to a large extent.

The effect of changing the internal pressure initial phase \( \varphi \) on vibration movements in the pipe is illustrated by the diagrams in Figures 12, 16, 17 (aqueous medium, \( \Omega = 6.8 \text{ rad/s} \)) and 29, 32, 33 (air medium, \( \Omega = 10.8 \text{ rad/s} \)) constructed for static pressure \( P_0 = 5.08 \text{ MPa} \). Changes that occur in the amplitudes of rotary and flexural vibrations are inconsiderable for aqueous and air media, along which the pipe
executes its movements. A slight increase can be noted in the frequency of flexural beatings in the pipe occurring with the growing magnitude of the initial phase.
Figure 3. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_a=0.01$ MPa, $\mu=24$ Pa·s, $\rho_{os}=800$ kg/m$^3$, $\Omega=6.8$ rad/s, $\varphi=0$ rad.
Figure 4. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_a=0.01$ MPa, $\mu=24$ Pa·s, $\rho_{os}=800$ kg/m$^3$, $\Omega=10.8$ rad/s, $\varphi=0$ rad.
Figure 5. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_\sigma=0.05$ MPa, $\mu=24$ Pa·s, $\rho_{\text{oil}}=800$ kg/m$^3$, $\Omega=2.8$ rad/s, $\varphi=0$ rad.
Figure 6. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_a=0.05$ MPa, $\mu=24$ Pa·s, $\rho_{os}=800$ kg/m$^3$, $\Omega=6.8$ rad/s, $\varphi=0$ rad.
Figure 7. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0 \text{ MPa}$, $P_a=0.05 \text{ MPa}$, $\mu=24 \text{ Pa}\cdot\text{s}$, $\rho_{\text{os}}=800 \text{ kg/m}^3$, $\Omega=10.8 \text{ rad/s}$, $\varphi=0 \text{ rad}$. 
Figure 8. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.01$ MPa,

$\mu = 24 \text{ Pa}\cdot\text{s}$, $\rho_m = 800 \text{ kg/m}^3$, $\Omega = 2.8 \text{ rad/s}$, $\varphi = 0 \text{ rad}$. 
Figure 9. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=5.08$ MPa, $P_a=0.01$ MPa, $\mu=24$ Pa/s, $\rho_0=800$ kg/m$^3$, $\Omega=6.8$ rad/s, $\varphi=0$ rad.
Figure 10. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.01$ MPa, $\mu = 24$ Pa·s, $\rho_{so} = 800$ kg/m$^3$, $\Omega = 10.8$ rad/s, $\varphi = 0$ rad.
Figure 11. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 24$ Pa·s, $\rho_{ox} = 800$ kg/m$^3$, $\Omega = 2.8$ rad/s, $\varphi = 0$ rad.
Figure 12. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 24$ Pa·s, $\rho_o = 800$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = 0$ rad.
Figure 13. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08 \text{ MPa}$, $P_a = 0.05 \text{ MPa}$, $\mu = 24 \text{ Pa s}$, $\rho_{os} = 800 \text{ kg/m}^3$, $\Omega = 6.8 \text{ rad/s}$, $\varphi = 0 \text{ rad}$. The Archimedes buoyancy force is not taken into account.
Figure 14. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_0 = 0.05$ MPa, $\mu = 24$ Pa·s, $\rho_{os} = 800$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = 0$ rad. The Coriolis inertia forces are not taken into account.
Figure 15. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, 
$\mu = 24$ Pa$s$, $\rho_{os} = 800$ kg/m$^3$, $\Omega = 10.8$ rad/s, $\varphi = 0$ rad.
Figure 16. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 24$ Pa·s, $\rho_{\text{os}} = 800$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = \pi / 2$ rad.
Figure 17. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 24$ Pa·s, $\rho_{os} = 800$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = \pi$ rad.
Figure 18. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1,0$ MPa, $P_a=0,01$ MPa, $\mu = 0,021$ Pa·s, $\rho_{os}=1,25$ kg/m$^3$, $\Omega =2,8$ rad/s, $\varphi = 0$ rad.
Figure 19. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_a=0.01$ MPa, $\mu = 0.021$ Pa·s, $\rho_{os}=1.25$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = 0$ rad.
Figure 20. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1,0$ MPa, $P_{\alpha}=0,01$ MPa, $\mu=0,021$ Pasc, $\rho_{\alpha}=1,25$ kg/m$^3$, $\Omega=10,8$ rad/s, $\varphi = 0$ rad.
Figure 21. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_a=0.05$ MPa, $\mu=0.021$ Pa·s, $\rho_0=1.25$ kg/m$^3$, $\Omega=2.8$ rad/s, $\varphi=0$ rad.
Figure 22. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1.0$ MPa, $P_a=0.05$ MPa,
$\mu=0.021$ Pa·s, $\rho_{os}=1.25$ kg/$m^3$, $\Omega=6.8$ rad/s, $\varphi=0$ rad.
Figure 23. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=1,0$ MPa, $P_a=0,05$ MPa, $\mu=0,021$ Pa s, $\rho_{os}=1,25$ kg/m$^3$, $\Omega=10,8$ rad/s, $\varphi=0$ rad.
Figure 24. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.01$ MPa, $\mu = 0.021$ Pa$\cdot$s, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 2.8$ rad/s, $\varphi = 0$ rad.
Figure 25. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.01$ MPa, $\mu = 0.021$ Pa·s, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = 0$ rad.
Figure 26. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.01$ MPa, $\mu = 0.021$ Pars, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 10.8$ rad/s, $\varphi = 0$ rad.
Figure 27. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 0.021$ Pa·s, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 2.8$ rad/s, $\varphi = 0$ rad.
Figure 28. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 0.021$ Pa·s, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 6.8$ rad/s, $\varphi = 0$ rad.
Figure 29. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 0.021$ Pa·s, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 10.8$ rad/s, $\varphi = 0$ rad.
Figure 30. Dependencies of the angle of rotation \( \theta \) and deflection \( w_0 \) of the middle point of the pipe span on the time \( t \) at \( P_0 = 5.08 \) MPa, \( P_a = 0.05 \) MPa, \( \mu = 0.021 \) Pa\( \cdot \)s, \( \rho_{os} = 1.25 \) kg/m\(^3\), \( \Omega = 10.8 \) rad/s, \( \varphi = 0 \) rad. The Archimedes buoyancy force is not taken into account.
Figure 31. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_d = 0.05$ MPa, $\mu = 0.021$ Pa·s, $\rho_{os} = 1.25$ kg/m$^3$, $\Omega = 10.8$ rad/s, $\varphi = 0$ rad. The Coriolis inertia forces are not taken into account.
Figure 32. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0=5.08$ MPa, $P_a=0.05$ MPa, 
$\mu=0.021$ Pa·s, $\rho_{os}=1.25$ kg/m$^3$, $\Omega=10.8$ rad/s, $\varphi=\pi/2$ rad.
Figure 33. Dependencies of the angle of rotation $\theta$ and deflection $w_0$ of the middle point of the pipe span on the time $t$ at $P_0 = 5.08$ MPa, $P_a = 0.05$ MPa, $\mu = 0.021$ Pa·s, $\rho = 1.25$ kg/m$^3$, $\Omega = 10.8$ rad/s, $\varphi = \pi$ rad.
Taking the above-said into account, we can make the following conclusions.

1. As the internal pressure static component increases, a simultaneous increase takes place in the amplitude of flexural vibrations and a decrease in the frequency of rotary vibrations of the pipe.

2. At one and the same value of static pressure, the largest changes in the amplitude of the pipeline’s rotary and flexural vibration movements are determined by changing values of circular frequency of the internal pressure dynamic component.

3. As is shown, if the pipe’s vibrations are executed in aqueous medium, the amplitudes of flexural and rotary vibrations in the pipe with and without account for drag forces differ to a large extent. If the pipe executes its vibrations in air medium, we can see a coincidence in the results of calculations with and without account for drag forces. In the latter case we can also observe a pronounced increase in the frequencies of both rotary and flexural vibrations of the pipe.

4. When the pipe moves in aqueous medium, a considerable increase takes place in the amplitude of flexural vibrations and a slight increase in the amplitude of rotary vibrations with an increase in the amplitude of internal pressure dynamic component. When the pipe moves in air medium, an increase in the amplitude of the internal pressure dynamic component leads to an increase in the amplitudes of both flexural and rotary vibrations.

5. It has been found that if we do not take into account the Archimedes buoyancy force, an essential increase in the frequencies and a decrease in the amplitudes of rotary and flexural vibrations occur simultaneously when the pipe moves in aqueous medium. In this case we can also note a considerable increase in the frequency of flexural beatings. When the pipe moves in air medium, its frequencies and amplitudes of rotary and flexural vibrations have no considerable changes.

6. If the Coriolis forces are not taken into account, both frequencies and amplitudes of rotary and flexural vibrations increase inconsiderably when the pipeline moves in aqueous medium. In this case flexural vibrations with the greatest amplitude continue for a shorter period of time. There is also a slight increase in the frequency of flexural beatings. When the pipe moves in air medium, its amplitudes and frequencies of rotary and flexural beatings are decreased to a considerable extent.

7. For aqueous and air media, in which the pipe executes its movements, changes in the amplitudes of its rotary and flexural vibrations are inconsiderable with varying values of the internal pressure initial phase. We can note a slight increase in the frequency of flexural beatings of the pipe with growing value of the initial phase.
References

ПРОСТРАНСТВЕННЫЕ ПАРАМЕТРИЧЕСКИЕ КОЛЕБАНИЯ ТРУБОПРОВОДА ПОД ДЕЙСТВИЕМ ПЕРЕМЕННОГО ВНУТРЕННЕГО ДАВЛЕНИЯ.
ИССЛЕДОВАНИЕ ВЛИЯНИЯ ХАРАКТЕРИСТИК ВНУТРЕННЕГО ДАВЛЕНИЯ В ЖИДКОСТИ, СИЛЫ АРХИМЕДА, СИЛ ИНЕРЦИИ КОРИОЛИСА И СИЛ СОПРОТИВЛЕНИЯ НА ИЗГИБНЫЕ И ВРАЩАТЕЛЬНЫЕ КОЛЕБАТЕЛЬНЫЕ ДВИЖЕНИЯ ТРУБОПРОВОДА. Ч. II

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Аннотация. Построена приближенная математическая модель изгибно-вращательных колебательных движений изогнутого собственным весом двухопорного трубопровода, разработанная в предположении малости упругости опор и деформаций трубы, связанных с ее выходом из плоскости изгиба, и учитывающая выталкивающую силу Архимеда, силы инерции Кориолиса, а также силы сопротивления пропорциональные первой степени скорости. Получены формулы, определяющие собственные частоты изгибно-вращательных колебаний трубопровода. Установлено влияние силы Архимеда, силы инерции Кориолиса, сил сопротивления, геометрических и физико-механических параметров трубы на ее собственные колебания.

Настоящая статья, являющаяся продолжением работы [1], посвящена решению задачи о пространственных изгибно-вращательных колебаниях трубопровода под действием переменного внутреннего давления. Особое внимание уделено исследованию влияния величин статической и динамической составляющих, круговой частоты и начальной фазы давления в жидкости, выталкивающей силы Архимеда, силы инерции Кориолиса и сил сопротивления на изгибные и вращательные колебательные движения трубопровода.

Ключевые слова: пространственные параметрические колебания, трубопровод, переменное внутреннее давление, статическое давление, начальная фаза, амплитуда, круговая частота изменения давления.
Литература


3. Тарг С.М. Основные задачи теории ламинарных течений. М: Гостехиздат, 1951, 420 с.


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