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NEW INFLOW PERFORMANCE RELATIONSHIP FOR SOLUTION-GAS DRIVE OIL RESERVOIRS

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Abstract. *The Inflow Performance Relationship (IPR) describes the behavior of the well's flowing pressure and production rate, which is an important tool in understanding the reservoir/well behavior and quantifying the production rate. The IPR is often required for designing well completion, optimizing well production, nodal analysis calculations, and designing artificial lift. Different IPR correlations exist today in the petroleum industry with the most commonly used models are that of Vogel's and Fetkovitch's. In addition to few analytical correlations, that usually suffers from limited applicability.*

In this work, a new model to predict the IPR curve was developed, using a new correlation that accurately describes the behavior the oil mobility as a function of the average reservoir pressure. This new correlation was obtained using 47 actual field cases in addition to several simulated tests.

After the development of the new model, its validity was tested by comparing its accuracy with that of the most common IPR models such as Vogel, Fetkovitch, Wiggins, and Sukarno models. Twelve field cases were used for this comparison. The results of this comparison showed that: the new developed model gave the best accuracy with an average absolute error of 6.6 %, while the other common models are ranked, according to their accuracy in the following order to be Fetkovich, Sukarno, Vogel, and Wiggins, with average absolute errors of 7 %, 12.1 %, 13.7 %, and 15.7 respectively.

The new developed IPR model is simple in application, covers wide range of reservoir parameters, and requires only one test point. Therefore, it provides a considerable advantage compared to the multipoint test method of Fetkovich. Moreover, due to its accuracy and simplicity, the new IPR provides a considerable advantage compared to the widely used method of Vogel.

Finally, the application of the new model is illustrated with field examples for current and future IPR computations.

Keywords*: Inflow Performance Relationship, oil mobility, solution- gas drive oil reservoir, empirically IPR correlations, analytically IPR correlations*

Introduction

For slightly compressible fluids, the productivity index is given by:

$$J=\frac{0.00708\cdot kh}{\ln\left(r_e/r_w\right)-0.75+S}\left[\frac{k_{ro}}{\mu_o B_o}\right]. \qquad (1)$$

Therefore, the variables that affecting the productivity index and in turn the inflow performance are the pressure dependent parameters (μ_o, B_o, and k_{ro}). Fig. 1 schematically illustrates the behavior of those variables as a function of average reservoir pressure (p_r). Above the bubble-point pressure (p_b), k_{r0} equals unity and the term ($k_{ro}/\mu_o B_o$) is almost constant. As the pressure declines below p_b, the gas is released from solution which can cause a large decrease in both k_{ro} and ($k_{ro}/\mu_o B_o$).

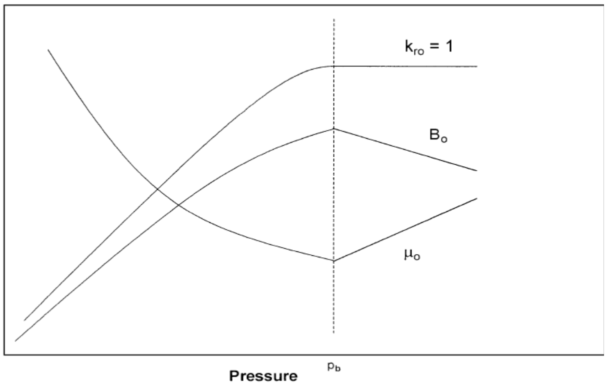

Fig. 1. Effect of p_r on B_o, μ_o, and k_{ro} (After Ahmed,T.[1])

Assuming that the well's productivity index (PI) is constant, the oil flow rate can be calculated as:

$$q_o = J\left(p_r - p_{wf}\right) . \tag{2}$$

Eq. 2 suggests that the inflow into a well is directly proportional to the pressure drawdown (Δp). Evinger and Muskat (1949) observed that as the pressure drops below p_b the inflow performance curves deviates from the simple straight-line relationship as shown in Fig. 2. Therefore, the above relationship is not valid for two-phase flow or in case of solution gas drives reservoirs.

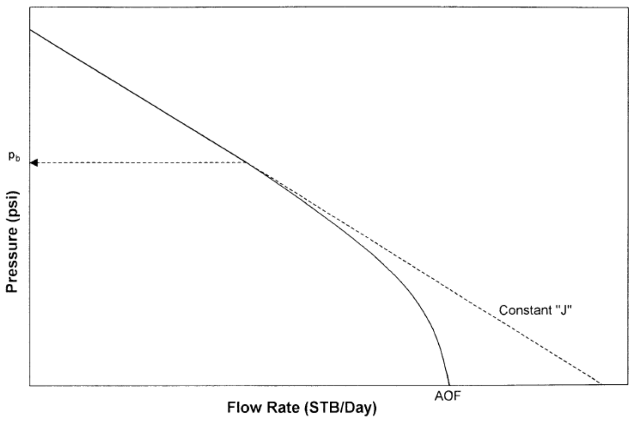

Fig. 2. The inflow performance curve below the bubble-point pressure (After Ahmed, T.[1])

Many IPR correlations addressed the curvature in Fig. 2 of the inflow performance curves in case of solution gas drive oil reservoirs in which p_b is the initial reservoir pressure. Based on the literature survey, the most known IPR correlations can be subdivided into empirically and analytically derived correlations. Some of the most known empirical correlations are Vogel (1968), Fetkovich (1973), Kilns and Majcher (1992), Wiggins (1993), and Sukarno et al. (1995). Some of the most known analytical correlations are Wiggins et al. (1991, 1992), and Del Castillo, et al. (2003).

The empirical derived correlations

Vogel's method

Vogel (1968) used a computer program based on Weller's (1966) assumptions and twenty-one reservoir data sets to develop an IPR as:

$$\frac{q_o}{q_{o,\max}}=1-0.2\left[\frac{p_{wf}}{p_r}\right]-0.8\left[\frac{p_{wf}}{p_r}\right]^2 . \tag{3}$$

Vogel's correlation gave a good match with the actual well inflow performance at early stages of production but deviates at later stages of the reservoir life. Therefore, this will affect the prediction of inflow performance curves in case of solution gas drive reservoirs, because at later stages of production the amount of the free gas that comes out of the oil will be greater than the amount at the early stages of production.

Fetkovich's method

Fetkovich (1973) developed an IPR based on multi-rate tests "40 different oil wells from six fields" and the general treatment of the inflow performance provided by Raghavan (1993) under pseudo-steady state conditions. Eq. 4 gives the oil flow rate as introduced by Raghavan (1993):

$$q_o=\bar{J}\int_{p_{wf}}^{p_r}\frac{k_{ro}(S_o)}{\mu_o B_o}dp , \tag{4}$$

where $\bar{J}$ is defined by:

$$\bar{J}=\frac{1}{1.142}\frac{kh}{\ln(r_e/r_w)-0.75+S} . \tag{5}$$

Eq. 4 is not useful in a practical sense, therefore, Fetkovich proposed the following relationship between the oil mobility function and p_r:

$$\frac{k_{ro}(S_o)}{\mu_o B_o}=x\cdot p_r , \tag{6}$$

where x is constant. Finally, the "Fetkovich form" of the IPR equation is given as the "backpressure" modification form, which is written as:

$$\frac{q_o}{q_{o,\max}}=\left[1-\frac{p_{wf}^2}{p_r^2}\right]^n . \tag{7}$$

Eq. 7 requires a multi-rate test to determine the value of *n*. As indicated, the main parameter that affect on the Fetkovich's model is the oil mobility as a function of p_r, which assumed to be linear relationship as illustrated in Fig. 3.

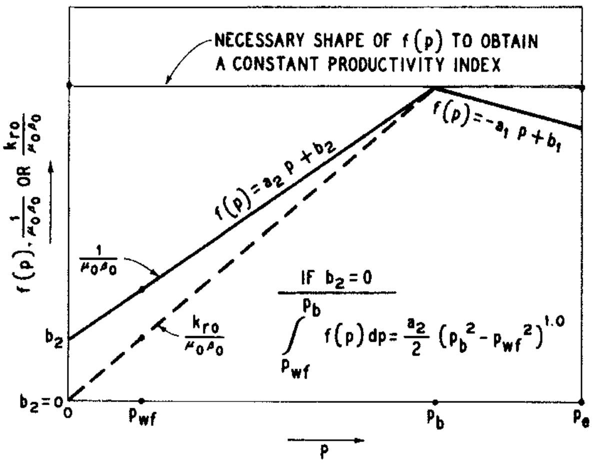

Fig. 3. Mobility-pressure behavior for a solution gas drive reservoir (After Fetkovich[4])

Klins and Majcher's method

Based on Vogel's work, Klins and Majcher (1992) developed the following IPR that takes into account the change in bubble-point pressure and reservoir pressure.

$$\frac{q_o}{q_{o,\max}|_{S=0}}=\left[1-0.295\left(\frac{p_{wf}}{p_r}\right)-0.705\left(\frac{p_{wf}}{p_r}\right)^{N_1}\right], \tag{8}$$

where $$N_1=\left(0.28+0.72\frac{p_r}{p_b}\right)\cdot\left(1.235+0.001\,p_b\right). \tag{9}$$

Wiggins's method

Wiggins (1993) developed the following generalized empirical three phase IPR similar to Vogel's correlation based on his developed analytical model in 1991:

$$\frac{q_o}{q_{o,\max}}=1-0.519167\left(\frac{p_{wf}}{p_r}\right)-0.481092\left(\frac{p_{wf}}{p_r}\right)^2 . \tag{10}$$

Sukarno and Wisnogroho's method

Sukarno and Wisnogroho (1995) developed an IPR (Eq. 11) based on simulation results that attempts to account for the flow-efficiency variation caused by rate-dependent skin:

$$\frac{q_o}{q_{o,\max}|_{S=0}}=FE\left[1-0.1489\left(\frac{p_{wf}}{p_r}\right)-0.4416\left(\frac{p_{wf}}{p_r}\right)^2-0.4093\left(\frac{p_{wf}}{p_r}\right)^3\right], \tag{11}$$

where:

$$FE = a_o + a_1\left(\frac{p_{wf}}{p_r}\right) + a_2\left(\frac{p_{wf}}{p_r}\right)^2 + a_3\left(\frac{p_{wf}}{p_r}\right)^3; \quad (12)$$

$$a_i = b_{oi} + b_{1i}\cdot S + b_{2i}\cdot S^2 + b_{3i}\cdot S^3. \quad (13)$$

In Eq. 13, a_i, b_{oi}, b_{1i}, b_{2i}, and b_{3i} are the fitting coefficients.

The analytical derived correlations

Wiggins's method

Wiggins (1991) and Wiggins, et al. (1992) studied the three-phase (oil, water, and gas) inflow performance for oil wells in a homogeneous, bounded reservoir. They started from the basic principle of mass balance with the pseudo-steady state solution to develop the following analytically IPR:

$$\frac{q_o}{q_{o,\max}} = 1 + \frac{C_1}{D}\left(\frac{p_{wf}}{p_r}\right) + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4. \quad (14)$$

Where, C_1, C_2, C_3...C_n, and D coefficients are determined based on the oil mobility function and its derivatives taken at p_r.

Wiggins, et al. (1991, 1992) found that the main reservoir parameter that plays a major role in the inflow performance curve is the oil mobility function. The major problem in applying this IPR is its requirement for the mobility derivatives as a function of p_r, which is very difficult in practice. Therefore, in 1993 Wiggins developed an empirical IPR (i.e., Eq.10) from this analytical IPR model by assuming a third degree polynomial relationship between the oil mobility function and p_r. Wiggins, et al. also presented plots of the oil mobility as a function of p_r taken at various flow rates. An example of the oil mobility-pressure profile that is presented by Wiggins (1991) is shown in Fig. 4.

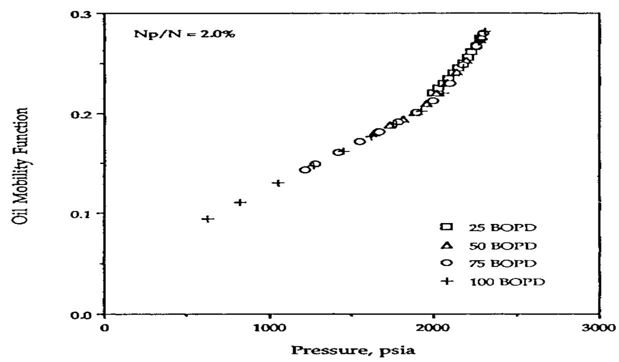

Fig. 4. The oil mobility profiles as a function of pressure - various flow rates (Case 2, after Wiggins, et al.[8])

Del Castillo's method

Del Castillo (2003), Del Castillo et al. (2003) developed theoretical attempt to relate the IPR with the fundamental flow theories. In this model, a second-degree polynomial IPR is obtained with a variable coefficient (v), or the oil IPR parameter that in fact be a strong function of pressure and saturation. The starting point for this development is the pseudo-pressure formulation for the oil phase, which is given as:

$$p_{p_o}(p) = \left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n} \cdot \int_{p_{base}}^{p} \frac{k_{ro}}{\mu_o B_o} dp \,. \quad (15)$$

In that work, the authors presumed that the oil mobility function has a linear relationship with the average reservoir pressure as given below:

$$\frac{k_{ro}}{\mu_o B_o}\Big|_{p_r} = f(p_r) = e + 2d \cdot p_r \,. \quad (16)$$

Where e, and d are constants established from the presumed behavior of the oil mobility profile. Fig. 3 refers to the physical interpretation of Eq. 16. Substituting with Eq. 16 in Eq. 15 and manipulating, the following equation could be presented:

$$\frac{q_o}{q_{o,\max}} = 1 - v\left[\frac{p_{wf}}{p_r}\right] - (1-v)\left[\frac{p_{wf}}{p_r}\right]^2 . \quad (17)$$

Specifically, the ν-parameter is given as:

$$v = \frac{1}{\left(1 + \frac{d}{e} p_r\right)} \,. \quad (18)$$

Wiggins (1991) and Del Castillo (2003) relationships can only be applied indirectly or inferred, by estimating the oil mobility as a function of pr to construct the IPR curve.

Summary of literature survey

As indicated, the empirical IPRs suffer from the limitation of their application range as they depend on the data used in their generation, and lack of accuracy. In addition, they aren't explicitly functioning of reservoir rock and fluid data, which are different from one reservoir to another. On the other hand, the analytical IPRs suffer from their difficulty to be applied due to its requirement to the oil mobility profiles with its derivatives, and the assumptions used in their development. As discussed, the main parameter that affects PI and in turns the IPR curves is the oil mobility as a function of pr. Therefore, the relationship between the oil mobility and pr should be accurately determined. In addition, the most common equation that represents a basic start point for the development of any IPR in case of solution gas drive reservoirs is Eq. 15, which mainly a function of the oil mobility (kro/μoBo).

Most of the empirical IPRs did not take into their consideration the whole effect of the oil mobility function, this in turn largely reduce the accuracy and utility of these

IPRs. Even though the models that considered this effect, such as the models of Fetkovich (1973) and Wiggins (1993), assumed the relationships between this function and pr, as a linear form and a third polynomial form for Fetkovich and Wiggins, respectively. In fact, these linear and polynomial forms don't accurately describe the behavior of the oil mobility as a function of pr with an accurate manner. On the other hand, some of analytical IPRs didn't considered the effect of oil mobility, except the models of Wiggins, et al. (1991, 1992) Del Castillo, et al. (2003). Wiggins's model is so complicated because it requires the oil mobility represented in its derivates as a function of pr, this is greatly difficult in application. Del Castillo's model is not accurate; this is because Del Castillo assumed a liner relationship between the oil mobility function and pr, which in turn reduce the accuracy of this model.

Another parameter should be considered in the selecting of the IPR method, is the aspect of conducting the flow tests. It is evident that test costs have to be taken into consideration. Finally, the range of applicability will also affect the selecting of the IPRs to predict the well performance.

Accordingly based on the literature survey in this work, it is necessary to:

– Develop a new, more general, simple, and consistent method to correlate inflow performance trends for solution gas drive oil reservoirs. This new method takes into consideration the behavior of the oil mobility function with the average reservoir pressure without the direct knowledge of this behavior.

– Determine the applicability and accuracy of the proposed new model by applying it on different field cases with a comparison with some of the most known and used IPR equations, considering a wide range of fluid, rock, and reservoir characteristics.

– Test some of the available IPR methods on field data.

– Address the prediction of future performance from current test information.

The new developed IPR model

In this work, a single well 3D radial reservoir model using MORE (2006) reservoir simulator was built. The reservoir simulation was used to investigate the shape and in turn the relationship between the oil mobility function and pr. Then, a new IPR was derived based on the resulted oil mobility-pressure profile; this new IPR is mainly a function of the relationship between the oil mobility and pr. Then, forty-seven field cases (published cases) were used to develop an empirical relationship between the oil mobility and pr. Thus, obtaining a new IPR model that is explicitly functioning of the oil mobility that is highly affecting the IPR model.

Mobility-reservoir pressure relationship

Production rate and pressure results from six simulation cases were used to develop the inflow performance curves. Table 1 presents the ranges of reservoir, rock, and fluid parameters used in the six simulation cases. The saturation and pressure information was also used to develop the mobility function profiles. The general simulation assumptions that were used in building the reservoir model can be summarized as follows:

- 3D radial flow into the well bore;
- the reservoir initially at the bubble point pressure;
- vertical well at the center of the formation;
- the well is completed through the whole formation thickness;
- homogeneous, bounded reservoir;
- isothermal conditions exist;
- no initial O.W.C. exist;
- capillary pressure is neglected;
- interfacial tension effects and non-Darcy flow effects are not considered.

Table 1. Ranges of reservoir parameters – simulation cases

Rock/Fluid Property	**Range**	**Units**
Average reservoir pressure, p_r	2000 - 5000	psia
Bubble point pressure, p_b	2000 - 5000	psia
Reservoir temperature, T	100 - 300	° F
Oil specific gravity relative to water, γ_o	0.7 - 0.85	dimensionless
Gas specific gravity relative to air, γ_g	0.5 - 1.2	dimensionless
Water specific gravity, γ_w	1.0 - 1.25	dimensionless
Water viscosity, μ_w	0.1 - 1.0	cp
Initial solution gas oil ratio, R_{soi}	0.47 - 2.16	Mcf/STB
Initial oil formation volume factor, B_{oi}	1.12 - 2.52	bbl/STB
Initial oil viscosity, μ_{oi}	0.09 - 0.44	cp
Z-factor	0.7 - 1.042	dimensionless
k_{rw} @ (S_{or})	0.1 - 0.4	fraction
k_{ro} @ (S_{wc})	0.2 - 1.0	fraction
k_{rg} @ $(1-S_{wc}-S_{or})$	0.3 - 1.0	fraction
Reservoir radius, r_e	100 - 10000	ft
Formation thickness, h	10 - 1000	ft
Absolute permeability, k	0.5 - 500	md

Results of the simulator

Fig. 5 shows the behavior of the mobility as a function of the pressure at different values of the flow rate during the two-phase flow for simulation case No.1 (Case-S1). The other five cases are shown in Fig. 5.1 through Fig. 5.5. The curves are typical of trend in addition to Fig. 5. Therefore, based on the six simulation cases, a reciprocal relationship between the oil mobility function and the average reservoir pressure was assumed and gives an acceptable and good match with the calculated simulator data as shown in Fig. 6.

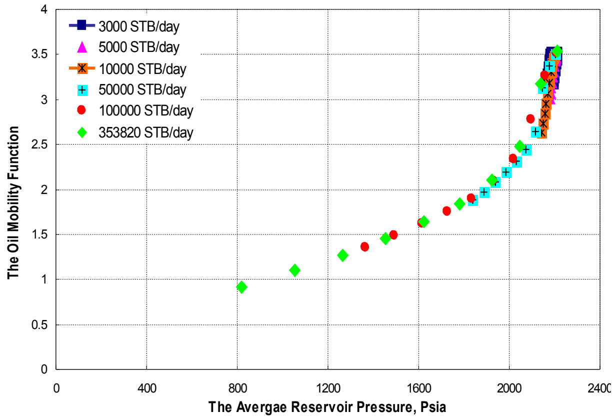

Fig. 5. Mobility-pressure behavior for solution gas drive oil reservoir – Case S1

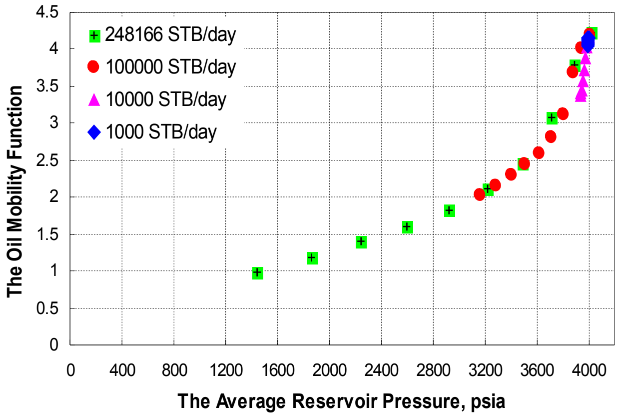

Fig. 5.1. Mobility-pressure behavior for solution gas drive oil reservoir – Case S2

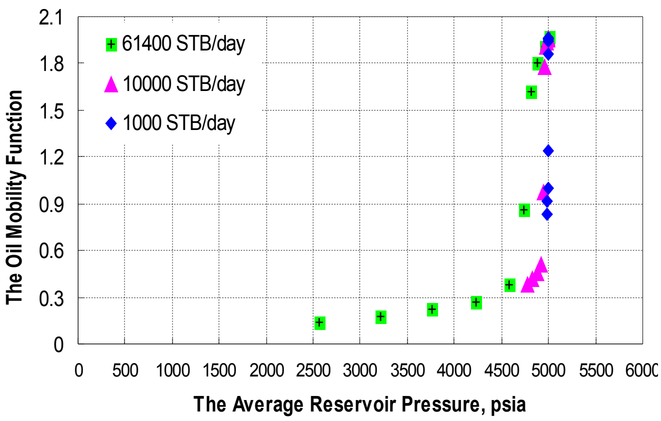

Fig. 5.2. Mobility-pressure behavior for solution gas drive oil reservoir – Case S3

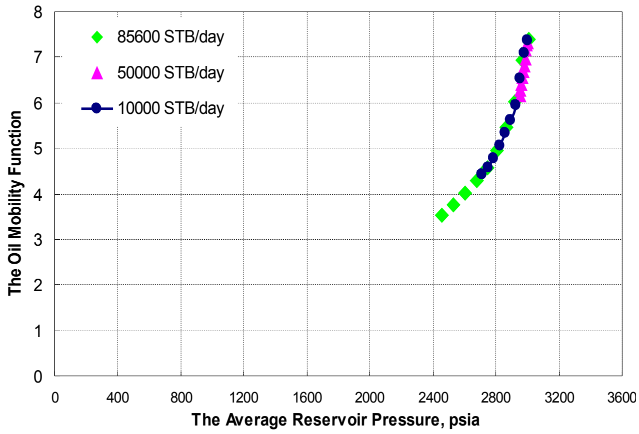

Fig. 5.3. Mobility-pressure behavior for solution gas drive oil reservoir – Case S4

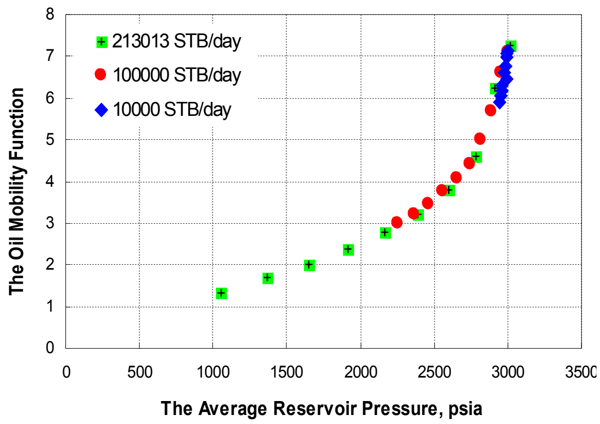

Fig. 5.4. Mobility-pressure behavior for solution gas drive oil reservoir – Case S5

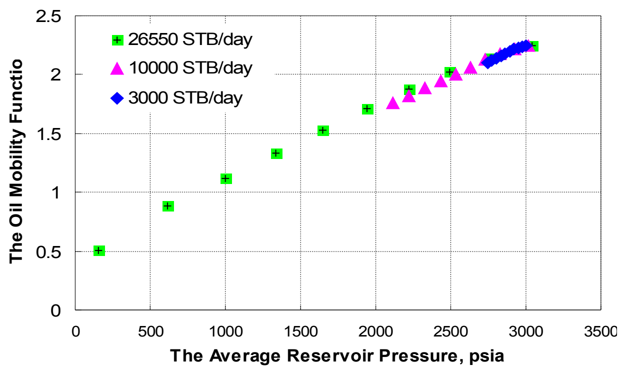

Fig. 5.5. Mobility-pressure behavior for solution gas drive oil reservoir – Case S6

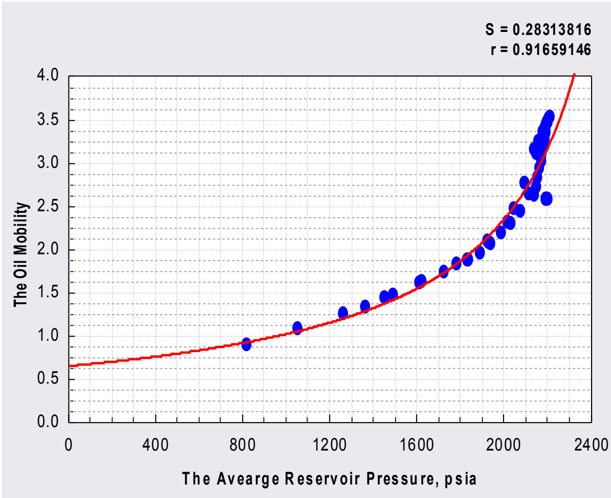

Fig. 6. Reciprocal model – Case S1

Derivation of the new IPR equation

The starting point for the derivation is the definition of the oil-phase pseudopressure for a single well in a solution gas drive reservoir and the pseudo-steady state flow equation for the oil-phase.

In this work, a new form for the oil mobility function at different values of the average reservoir pressure (i.e., the reciprocal relationship) is obtained from the simulation study that performed on MORE simulators using the six simulation cases. This reciprocal relationship was used as:

$$\left[\frac{k_{ro}}{\mu_o B_o}\right]_{p_r} = \frac{1}{x \cdot p_r + y}, \tag{19}$$

where x and y are constants established from the presumed behavior of the mobility profile.

Substituting Eq. 19 in Eq. 15 and manipulating (the details are provided in Appendix A), the following new IPR equation will be introduced:

$$\frac{q_o}{q_{o,\max}} = 1 - \frac{\ln(\alpha \cdot p_{wf} + 1)}{\ln(\alpha \cdot p_r + 1)}, \tag{20}$$

where: α is the oil IPR parameter for the new IPR model and on the same time it is represents the two constants x and y that shown in Eq. 19 *($\alpha = x/y$).*

Eq. 20 is the proposed new IPR model. As recognized, the α-parameter is not "constant," therefore; forty-seven field cases (published cases) were used to develop a generalized chart between the α-parameter and p_r (i.e. Fig. 7). Table 2 introduces the ranges of data used in the development of these two equations.

Fig. 8 showing the fitting results with the used 47 field cases. As we can see, the fitting line can be divided into two fitting trends to obtain the best accuracy, and we found by trial and error that p_r threshold is 1600 psi.

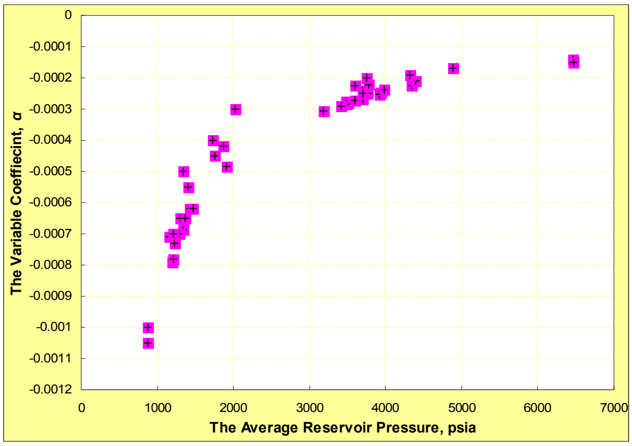

Fig. 7. The variable coefficient α and the average reservoir pressure chart

Table 2. Ranges of data used in the development α-p_r relationship

Properties	Data range
1. Fluid properties data:	
Gas specific gravity	0.60 - 0.8
API gravity of oil	20 - 60
Water specific gravity	1.04 - 1.074
Initial oil formation volume factor (bbl/STB)	1.3 - 1.94
Initial oil viscosity (cp)	0.27 - 0.99
Initial solution gas oil ratio,(Mscf/STB)	0.132 - 4.607
Bubble point pressure (psia)	Up to 7000
2. Rock properties data:	
Porosity	0.1 - 0.35
Absolute permeability (md)	2.5 - 2469
Irreducible water saturation	0.1 - 0.32
Residual oil saturation (W/O)	0.08 - 0.17
Residual oil saturation (G/O)	0.07 - 0.14
Critical gas saturation	0.02 - 0.17
Total compressibility (psi^{-1})	0.33 x 10^{-3} - 30 x 10^{-6}
Oil relative permeability @ 0.02 and 0.1 S_{gc}	0.444 - 0.52
3. Reservoir and well dimension:	
Average reservoir pressure (psia)	Up to 7000
Drainage area (acres)	20 - 80
Formation thickness (feet)	10 - 182
Reservoir radius (feet)	250 - 1053
Well bore radius (feet)	0.33 - 0. 35
Reservoir temperature (deg. F)	156 - 238

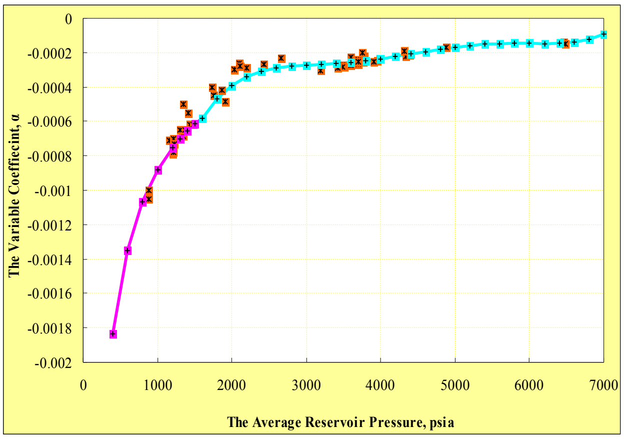

Fig. 8. The variable coefficient α and p_r fitting chart

Based on Fig.7 and Fig. 8, two empirical relationships of the variable coefficient α as function of p_r were developed as:

When p_r-range is less than or equal to 1600 psia the following relationship was developed:

$$\alpha = \frac{1}{a \cdot p_r + b}, \tag{21}$$

where: a = -0.981, and b = -152.585.

When p_r-range is greater than or equal to 1600 psia the following relationship was developed:

$$\alpha = c + d \cdot p_r + e \cdot p_r^2 + f \cdot p_r^3 + g \cdot p_r^4 + h \cdot p_r^5 . \tag{22}$$

As an important note; if we calculate the variable coefficient α at 1600 psi, it must be the same from Eq. 21 and Eq. 22 which increase the power and utility of the two developed α-empirical relationships. Table 3 shows the constants of Eq. 22.

Table 3. Constants of Eq. 22

Constant	Value
c	-0.0043065
d	4.98E-06
e	-2.41E-09
f	5.69E-13
g	-6.48E-17
h	2.85E-21

Table 2 introduces the ranges of data used in the development of Eq. 21 and Eq. 22. Finally, Eq. 20, Eq. 21, and Eq. 22 represent the new developed IPR model.

Methodology to use the new IPR model

Step 1. If p_r is less than or equal to 1600 psia, therefore, the oil IPR parameter (α) is calculated using Eq. 21 as follows:

$$\alpha = \frac{1}{-0.981 \cdot p_r - 152.585}.$$

If p_r is greater than or equal to 1600 psia, therefore, the oil IPR parameter (α) is calculated using Eq. 22 as follows:

$$\alpha = -0.0043065 + 4.98\text{E-}06 \cdot p_r - 2.41\text{E-}09 \cdot p_r^2 + 5.69\text{E-}13 \cdot p_r^3 - 6.48\text{E-}17 \cdot p_r^4 + 2.85\text{E-}21 \cdot p_r^5$$

Step 2. Calculate $q_{o,\max}$ using Eq. 20 at any given test point:

$$q_{o,\max} = q_o(test) / \left[1 - \frac{\ln(-\alpha \cdot p_{wf(test)} + 1)}{\ln(-\alpha \cdot p_r + 1)} \right] \text{ STB/day.}$$

Step 3 . Assume several values for p_{wf} and calculate the corresponding q_o using Eq. 20:

$$q_o = q_{o,\max} \cdot \left[1 - \frac{\ln(\alpha \cdot p_{wf} + 1)}{\ln(-\alpha \cdot p_r + 1)} \right] \text{ STB/day.}$$

Step 4. For future IPR, calculate α_f using the future value of $p_r)_f$ using Eq. 21 or Eq. 22 according to the value of $p_r)_f$:

Step 5. Solve for $q_{o,\,max,}$ at future conditions using Fetkovich's equation as follows:

$$q_{o,\max})_f = q_{o,\max})_p \cdot \left[\frac{p_r)_f}{p_r)_p} \right]^{3.0} \text{ STB/day.}$$

Step 6. Generate the future inflow performance-curve by applying Eq. 20 as follows:

$$q_o = q_{o,\max})_f \cdot \left[1 - \frac{\ln(\alpha_f \cdot p_{wf} + 1)}{\ln(\alpha_f \cdot p_r)_f + 1)} \right] \text{STB/day.}$$

Validation of the new IPR model

To verify and validate the new developed IPR model, information from 12 field cases were collected and analyzed to get the present inflow performance, and two field cases were collected and used to predict the future IPR curve. The field cases and reservoir data of these cases are included in Table 4. Each field case uses actual field data which representing different producing conditions. In order to test the accuracy and reliability of the new developed IPR model, which is single point method, it will be compared to some of the other two-phase IPR methods currently available in the industry. These methods are those of Vogel (single point method), Fetkovich (multi-point method), Wiggins (single point method), and Sukarno (single point method) for the present inflow performance. And, Vogel, Fetkovich, Wiggins for the future inflow performance

Elias (2009) presents the complete details of the comparison analysis while the cases analyzed for present and future performances are summarized in Table 4 and Table 5, respectively.

Table 4. Validation field cases analyzed for the present performance

Case	Case Name	Case Type	p_r, psia
1	Carry City Well	Vertical Well	1600
2	Well M110-1979	Vertical Well	2320
3	Well M200	Vertical Well	3263
4	Case X1B	Horizontal Well	2580
5	Well 1, Gulf of Suez, Egypt	Vertical Well	2020
6	Well 3-Field C	Vertical Well	3926
7	Well 4	Vertical Well, Layered Reservoir	5801
8	Well E, Keokuk Field	Vertical Well	1710
9	Well A, Keokuk Field	Vertical Well	1734
10	Well TMT-27	Vertical Well	868
11	Well A	Vertical Well	1785
12	Well 8, West Texas	Vertical Well	640

Table 5. Validation field cases analyzed for the future performance

Case	Test Chronology	Case Name	p_r, psia
1	Present	Well M110-1979	2321
1	Future	Well M110-1987	2067
2	Present	Well A, Keokuk Field-1934	1734
2	Future	Well A, Keokuk Field-1935	1609

Field case No. 1: Carry City well

Gallice et al. (1999) presented multirate-test data for a well producing from the Hunton Lime in the Carry City Field, Oklahoma. The test was conducted in approximately 2 weeks during the well, which was producing at random rates, rather than in an increasing or decreasing rate sequence. The average reservoir pressure was 1600 psia, with an estimated bubble-point pressure of 2530 psia and an assumed skin value of zero. The multi-rate test of this well is summarized in Table 6.

Table 7 presents the predictions of the well's performance for the test information at a flowing bottomhole pressure of 1194 psia, which representing a 25 % of the pressure drawdown. As can be observed, the maximum well deliverability varies from

2550 to 4265 STB/D. The largest flow rate was calculated with Wiggins's IPR, while the smallest rate was obtained using Fetkovich model. Fig. 9 shows the resultant IPR curves for the different methods of calculations such as Vogel, Fetkovich, Wiggins, and Sukarno in comparison with the actual field data and the new developed IPR model. It is clear from this figure that the method of the new developed IPR model is succeed to estimate the actual well performance. In addition, it can be clearly concluded from this figure that the methods of the new developed IPR model and Fetkovich's model are nearly estimate the maximum oil flow rate for this well more accurately than the other models, and as indicated, the other methods overestimate the actual performance.

Table 6. Test data- Field Case No.1

Test Data	
p_{wf}, psia	q_o, STB/D
1600	0
1558	235
1497	565
1476	610
1470	720
1342	1045
1267	1260
1194	1470
1066	1625
996	1765
867	1895
787	1965
534	2260
351	2353
183	2435
166	2450

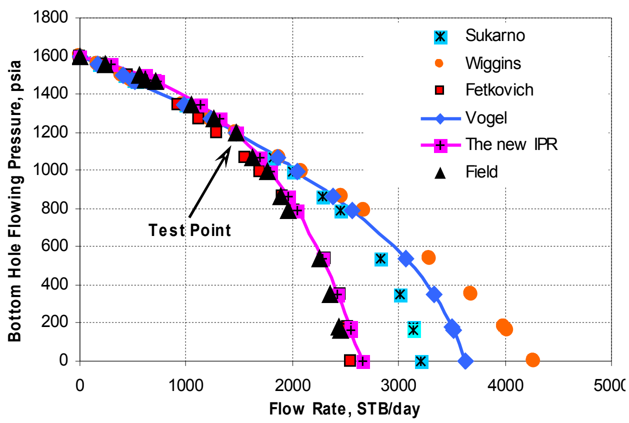

Fig. 9. The predicted inflow curves by the different used methods in comparison to the actual field data for Case No.1

Table 7. Prediction of the performance of Case No.1 at 25 % of the pressure drawdown

Field Data		**The new IPR method**	**Vogel method**	**Fetkovich method**	**Wiggins method**	**Sukarno method**
p_{wf}, **psia**	q_o, **STB/D**	q_o, **STB/D**	q_o, **STB/D**	q_o, **STB/D**	q_o, **STB/D**	q_o, **STB/D**
1600	0	0	0	0	0	0
1558	235	297	169	213	164	177
1497	565	614	408	444	398	423
1476	610	703	489	516	477	505
1470	720	728	511	536	499	528
1342	1045	1140	977	920	965	995
1267	1260	1321	1233	1115	1225	1244
1194	1470	1470	1470	1288	1470	1470
1066	1625	1688	1856	1559	1879	1828
996	1765	1790	2051	1690	2091	2003
867	1895	1954	2382	1905	2462	2290
787	1965	2044	2569	2021	2679	2445
534	2260	2284	3062	2309	3297	2830
351	2353	2428	3329	2447	3680	3018
183	2435	2544	3507	2522	3985	3133
166	2450	2555	3521	2527	4013	3142
0		2657	3627	2550	4265	3205

The average absolute errors percent between the actual flow-rate data and the calculated rate for the five IPR methods that used in this study are shown in Fig. 10 for the comparison. It is clear from this figure that the new developed IPR model has the lowest average absolute error percent that is 6.47 %, while the average absolute error percent for Fetkovich's method is 8.56 %. The other single-point methods have average absolute errors percent ranging from 20.1 to 32.3 % for Sukarno and Wiggins, respectively.

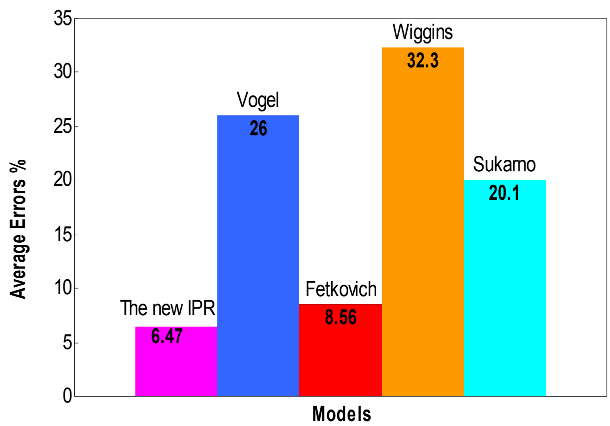

Fig. 10. The average absolute errors percent at 25 % drawdown for Case No.1

In summary, the new model provided the best estimates of well performance for this case's entire range of interest. The multipoint method of Fetkovich tends to do a better job of predicting well performance than the other three single-point methods. Overall, the single-point methods of Vogel, Wiggins, and Sukarno provided similar great average differences in this case. As indicated in this work, the more important relationship to evaluate well performance is the relationship between the oil mobility function and the average reservoir pressure, and this was clearly demonstrated from the value of the average absolute error percent that resulted from using the new developed IPR model.

Field cases summary for the present inflow performance

The additional cases and their analysis are presented in detail in Del Castillo (2003). Table 8 presents a summary of the average absolute errors percent that was obtained for each method in each one of the twelve case studies that were examined. As indicated, the method of the new developed IPR model always provided the most reliable estimates of the actual well data analyzed. It has the lowest value of the total average absolute error percent, which is 6.6 % in comparison with that of Fetkovich's method, which has a reasonable average absolute error percent of 7 % but is still higher than the method of the new developed IPR model. The other methods always provided less accurate values for the pressure-rate estimates of the actual well data that used in this analysis.

Table 8. Summary of the average absolute error percent – 12 cases

Case	**Total Average Errors %**				
	The new IPR	**Vogel**	**Fetkovich**	**Wiggins**	**Sukarno**
1	6.47	26	8.56	32.3	20.1
2	1	5.1	6.4	6.8	3.5
3	7.4	15.1	7.7	15.8	14.2
4	19.6	3	6.3	3.7	10.5
5	3.4	7	4.15	7.7	5.9
6	5.7	14	9.61	17.2	9.7
7	3.1	13	6.72	15.1	11.1
8	8	32	6	33.3	31
9	5.6	14.7	6	17.0	11.6
10	5.9	13.7	9.7	16.0	10.6
11	5.4	6	7.6	8.7	3.4
12	7.2	14	4.7	14.3	13.1
Average %	6.6	13.7	7	15.7	12.01.13

The method of the new developed IPR model tends to do a better job of predicting well performance than the other methods, and this it may be due to assume an accurate relationship between the oil mobility function and the average reservoir pressure (i.e., the

Reciprocal Relationship). Overall, the single-point methods of Vogel, Wiggins, and Sukarno provided great average absolute errors percent in the cases examined – 12.1 to 15.7 %.

However, the following comment should be introduced based on the above table Case No.4, the Vogel's model provided the best estimates of well performance for this case. The single point method of Wiggins tends to do a good job of predicting well performance in this case. Finally, it can be concluded from this case that the new IPR model has some limitations in case of low-pressure reservoirs, which have a reservoir pressure less than 1000 psia. This is because there were no sufficient data below 1000 psia in case of development the new IPR model, therefore it is recommended in this case to use Vogel's model.

Field case summary for the future performance

The analyses of the two future field cases are presented in detail in Elias (2009). Table 9 presents a summary of the average absolute errors percent that was obtained for each method in each one of the two future case studies that were examined. As indicated, the method of the new developed IPR model always provided the most reliable estimates of the actual well data analyzed. It has the lowest value of the total average absolute error percent, which is 15 %. The other methods always provided less accurate values for the pressure-rate estimates of the actual future well data that used in this analysis. Overall, the other methods of Fetkovich, Vogel, and Wiggins provided great average absolute errors percent in the cases examined range from 25.5 % and 31.5 % for Fetkovich and Vogel, respectively.

Table 9. Summary of the average absolute errors percent – Future field cases

Future Field Case	Total Average Errors %			
	The new IPR	Vogel	Fetkovich	Wiggins
1	21	28	36	23
2	9	35	15	38
Average %	15	31.5	25.5	30.5

However, general comment can be presented based on Table 9 and the future cases analyzed in this work, the average absolute error percent is almost as great for the Fetkovich's method as compared to the method of the new developed IPR model – 25.5 % compared to 15 %. As indicated, the main reasons for that are:

1. Fetkovich considered the relationship between the mobility function and the average reservoir pressure is linear relationship (see Fig. 3), but the new IPR model considered this relation is reciprocal relationship (see Fig. 5 and Fig. 6) which cover the entire range of interest more accurately.

2. The backpressure equation parameter (n) of Fetkovich IPR equation does not take into consideration the change in the average reservoir pressure.

3. The new model IPR parameter (α) of Eq. 20 takes into consideration the change in the average reservoir pressure (see Eq. 21 and Eq. 22).

Conclusions

In this work, we reviewed the most commonly used IPR models, also, we developed new IPR model. The new IPR was compared to the most commonly used models using field data (12 field cases). Based on this work, we can conclude the following:

1. A general correlation for α-parameter that represent the oil mobility as a function of p_r was developed by using 47 field cases. A new method to construct and predict the IPR curve for solution gas drive reservoirs was developed by using this general correlation of α- parameter

2. The validity of the new IPR model was tested through its application on 12 field cases in comparison with the behavior of the most common methods that are used in the industry. The results of this validation showed that the new IPR model ranked the first model that succeeded to predict the behavior of the IPR curve for the 12 examined field cases, while the other models of Fetkovich, Sukarno, Vogel, and Wiggins ranked the second, the third, the forth, and the fifth, respectively

3. The new IPR model requires one test point and is as accurate or more than Fetkovich's model which requires three test points

4. The new developed IPR outperformed all available IPR models except at low pressures (Less than 1000 psia). At these low pressures Vogel's correlation was found to be the most accurate model.

5. The range of applicability of Alpha- pressure relationships Eq. 21 and Eq. 22 is 860 to 7000 psi.

Nomenclature

A	Drainage area of well, sq ft
a_0, a_1, a_2, a_3	Constants for Sukarno and Wisnogroho, dimensionless
API	stock tank oil liquid gravity in ,^{O} API
b_0, b_1, b_2, b_3	Constants for Sukarno and Wisnogroho, dimensionless
B_o	Oil formation volume factor, bbl/STB
B_g	Gas formation volume factor, bbl/SCF
C_A	Shape constant or factor, dimensionless
C_1, C_2, C_3, C_4, D	Wiggins's constants, dimensionless
d, e	Del Castillo, Yanil's constants, dimensionless
h	Formation thickness, ft
J	Productivity index of the reservoir (PI), STB/psi
n	deliverability exponent for Fetkovich, dimensionless

N_1	Oil IPR parameter for Klins's equation, dimensionless
p_b	Bubble Point Pressure, psia
p_D	Dimensionless pressure
p_e	Pressure at the outer boundary, psia
PI_α	Productivity index from the new IPR model, STB/psi
p_r	Average reservoir pressure, psia
p_{wf}	Bottom hole flowing pressure, psia
q_o	Oil flow rate, STB/D
$q_{o,\,max}$	Maximum oil flow rate, STB/D
r_e	Drainage Radius, ft
R_s	Solution gas-oil ratio, scf/STB
r_w	Well Radius, ft
S	Radial flow skin factor, dimensionless
S_o	Oil saturation, fraction
T	Reservoir temperature, °F
x	Reciprocal model constant, dimensionless
y	Reciprocal model constant, dimensionless
$\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_D=0}$	Mobility ratio at zero dimensionless pressure
$\left(\frac{k_{ro}}{\mu_o B_o}\right)^{/}_{p_D=0}$	Mobility ratio first derivative at zero dimensionless pressure
$\left(\frac{k_{ro}}{\mu_o B_o}\right)^{//}_{p_D=0}$	Mobility ratio second derivative at zero dimensionless pressure
$\left(\frac{k_{ro}}{\mu_o B_o}\right)^{///}_{p_D=0}$	Mobility ratio third derivative at zero dimensionless pressure
α	Oil IPR parameter for the new IPR model, dimensionless
γ	Euler's constant (0.577216)
γ_g	Gas gravity, fraction
γ_o	Oil gravity, fraction
μ_o	Oil Viscosity, cp
Δp	Pressure drawdown, psi

References

1. Tarek Ahmed, Paul McKinney. Advanced reservoir engineering. Elsevier Inc., Gulf Professional Publishing, 2005. PP. 342-361.

2. Archer R.A., Castillo Y. Del, Blasingame T.A. New perspectives on vogel type ipr models for gas condensate and solution-gas drive systems, SPE paper 80907 presented at the *SPE Production operations symposium*, Oklahoma, OK, 23- 25 March 2003. DOI: 10.2118/80907-MS

3. Y. Del Castillo. New inflow performance relationships for gas condensate reservoirs. M.S. Thesis. Texas A& M University, May 2003, College Station.

4. Elias Mohamed. New inflow performance relationship for solution gas drive reservoirs. MS thesis, U. of Cairo, Giza, Egypt (2009).

5. Evinger H.H., Muskat J. Calculation of theoretical productivity factors, *Transactions AIME*, 1942, Vol. 146, pp. 126 - 139.

6. Fetkovich M.J. The isochronal testing of oil wells. SPE Paper 4529 presented at the *Fall Meeting of the Society of Petroleum Engineers of AIME*, Las Vegas, NV, Sep. 30-Oct. 3, 1973. DOI: 10.2118/4529-MS

7. Gallice F. A comparison of two-phase inflow performance relationships. MS thesis, U. of Oklahoma, Norman, Oklahoma (1997).

8. Frederic Gallice, Michael L. Wiggins. A comparison of two-phase inflow performance relationships. SPE Paper 88445 presented at *SPE Mid-Continent Operations Symposium*, 28-31 March 1999, Oklahoma City, Oklahoma. DOI: 10.2118/52171-MS

9. Klins M.A., Majher M.W. Inflow Performance relationships for damaged or improved wells producing under solution-gas drive. SPE paper 19852, *Journal of Petroleum Technology*, 1992, Vol. 44, Num. 12, pp. 1357 - 1363. DOI: 10.2118/19852-PA

10. MORE Manual, Version 6.3, ROXAR (2006).

11. Rajagopal Raghavan. Well test analysis. PTR Prentice Hall, 1993. PP. 513 - 514.

12. Sukarno P., Wisnogroho A. Generalized two-phase IPR curve equation under influence of non-linear flow efficiency, *Proc., Soc of Indonesian Petroleum Engineers Production Optimization Intl. Symposium*, 1995, Bandung, Indonesia, pp. 31 - 3.

13. Vogel J.V. Inflow performance relationships for solution gas drive wells, *Journal of Petroleum Technology*, 1968, Vol. 20, Num. 1, pp. 83 - 92. (SPE Paper 1476). DOI: 10.2118/1476-PA

14. Weller W.T. Reservoir performance during two-phase flow, *Journal of Petroleum Technology*, 1966, Vol. 18, Num. 2, pp. 240 - 246. (SPE Paper 1334).
DOI: 10.2118/1334-PA

15. Wiggins M.L, Russel J.L., Jennings J.W. Analytical inflow performance relationships for three phase flow, SPE Paper 24055 presented at *SPE Western Regional Meeting*, 30 March-1 April 1992, Bakersfield, California. DOI: 10.2118/24055-MS

16. Wiggins M.L., Generalized inflow performance relationships for three-phase flow, SPE Paper 25458 presented at the *SPE Production Operations Symposium*, Oklahoma City, OK, March 21-23 1993. DOI: 10.2118/25458-MS

17. Wiggins M.L., Wang H.-S. A two-phase IPR for horizontal oil wells, SPE Paper 94302 presented at *SPE Production Operations Symposium*, 16-19 April 2005, Oklahoma City, Oklahoma. DOI: 10.2118/94302-MS

18. Wiggins M.L. Inflow Performance of Oil Wells Producing Water, PhD dissertation, Texas A&M University, TX (1991).

Appendix A

In this Appendix, the derivation of the new IPR equation is based on the pseudo-steady state flow equation for a single well in a solution gas drive reservoir systems (pseudopressure formulation). In addition the relation between the mobility of the oil phase and p_r (i.e., Reciprocal relationship – $Mo = 1.0 / (a \cdot p_r + b)$) is used. Where a and b are the two equation variables. The definition of the oil-phase pseudopressure for a single well in a solution gas drive reservoir is given as:

$$p_{p_o}(p) = \left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n} \cdot \int_{p_{base}}^{p} \frac{k_{ro}}{\mu_o B_o} dp \,. \tag{A.1}$$

The pseudo-steady state flow equation for the oil-phase in a solution gas drive reservoir is given by:

$$p_{p_o}(p_r) = p_{p_o}(p_{wf}) + q_o \cdot b_{SS} \,, \tag{A.2}$$

where:

$$b_{SS} = 141.2 \frac{\mu_o B_o}{k_{ro}} |_{p_n} \left[\frac{1}{h}\left(\ln\left(\frac{r_e}{r_w} - \frac{3}{4} + S\right)\right)\right] . \tag{A.3}$$

In this work, a new form for the oil mobility function at different values of the average reservoir pressure (i.e., the reciprocal relationship) is obtained from the result of the simulation study that performed on MORE simulators using the six simulation cases as follows:

$$\left[\frac{k_{ro}}{\mu_o B_o}\right]_{p_r} = \frac{1}{x \cdot p_r + y} \,. \tag{A.4}$$

Where, x and y are two variables established from the presumed behavior of the mobility profile.

Solving Eq. A.2 for the oil rate, q_o, the following equation for the oil flow rate can be presented:

$$q_o = \frac{1}{b_{SS}}\left[p_{p_o}(p_r) - p_{p_o}(p_{wf})\right] . \tag{A.5}$$

Solving Eq. A.5 for the maximum oil rate, $q_{o,\,max}$ (i.e., at p_{wf} = zero or p_{po} (p_{wf}) = 0):

$$q_{o,\max} = \frac{1}{b_{SS}}\left[p_{p_o}(p_r) - p_{p_o}(p_{wf}=0)\right] . \tag{A.6}$$

Dividing Eq. A.5 by Eq. A.6 gives the "IPR" form (i.e., $q_o/q_{o,\,\max}$) in terms of the pseudopressure functions, which yields:

$$\frac{q_o}{q_{o,\max}} = \frac{p_{p_o}(p_r) - p_{p_o}(p_{wf})}{p_{p_o}(p_r) - p_{p_o}(p_{wf}=0)} \,. \tag{A.7}$$

At this point, it should be noted that, *it is not the goal* to proceed with the development of an IPR model in terms of the pseudopressure functions, $p_{po}(p)$-rather, the goal is to develop a simplified IPR model using Eq. A.4 and Eq. A.7 as the base relations. Substituting Eq. A.4 into Eq. A.1, this yields:

$$p_{p_o}(p)=\left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n}\cdot\int_{p_{base}}^{p}\left[\frac{1}{x\cdot p+y}\right]dp=\left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n}\frac{1}{a}\left[\ln(x\cdot p+y)\right]_{p_{base}}^{p}.$$

Or,

$$p_{p_o}(p)=\left[\frac{\mu_o B_o}{k_{ro}}\right]_{p_n}\frac{1}{a}\left[\ln(x\cdot p+y)-\ln(x\cdot p_{base}+y)\right].\tag{A.8}$$

Substituting Eq .A.8 into Eq. A.7, this gives:

$$\frac{q_o}{q_{o,\max}}=\frac{[\ln(x\cdot p_r+y)-\ln(x\cdot p_{base}+y)]-[\ln(x\cdot p_{wf}+y)-\ln(x\cdot p_{base}+y)]}{[\ln(x\cdot p_r+y)-\ln(x\cdot p_{base}+y)]-[\ln(y)-\ln(x\cdot p_{base}+y)]}$$

Or,

$$\frac{q_o}{q_{o,\max}}=\frac{\ln(x\cdot p_r+y)-\ln(x\cdot p_{wf}+y)}{\ln(x\cdot p_r+y)-\ln(y)}.\tag{A.9}$$

Rearranging Eq. A.9 gives the following form:

$$\frac{q_o}{q_{o,\max}}=\frac{\ln(x\cdot p_r+y)}{\ln(x\cdot p_r+y)-\ln(y)}-\frac{\ln(x\cdot p_{wf}+y)}{\ln(x\cdot p_r+y)-\ln(y)}.\tag{A.10}$$

Or,

$$\frac{q_o}{q_{o,\max}}=\frac{\ln(x\cdot p_r+y)}{\ln\left(\dfrac{x\cdot p_r+y}{y}\right)}-\frac{\ln(x\cdot p_{wf}+y)}{\ln\left(\dfrac{x\cdot p_r+y}{y}\right)}.\tag{A.11}$$

Dividing the right term through Eq. A.11 by the term y gives the following form:

$$\frac{q_o}{q_{o,\max}}=\frac{\ln\left(\dfrac{x}{y}p_r+1\right)}{\ln\left(\dfrac{x}{y}p_r+1\right)}-\frac{\ln\left(\dfrac{x}{y}p_{wf}+1\right)}{\ln\left(\dfrac{x}{y}p_r+1\right)}.\tag{A.12}$$

And then, replace *x/y* by α and substituting this definition into Eq. A.12, this yields the following IPR form:

$$\frac{q_o}{q_{o,\max}}=1-\frac{\ln(\alpha\, p_{wf}+1)}{\ln(\alpha\, p_r+1)}.\tag{A.13}$$

Where: α is the oil IPR parameter for the new IPR model.

It is suggested that Eq. A.13 serves as an equation of the proposed new IPR model.