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ON COMPUTING RELATIVE PERMEABILITIES

S.Z. Fatikhov

Tyumen Branch of the Khristianovich Institute
of Theoretical and Applied Mechanics of SB RAS, Tumen, Russia
e-mail: f_salih@mail.ru

V.R. Syrtlanov

"Gazpromneft STC" LLC, Saint-Petersburg, Russia

Abstract. *Relative phase permeability (RPP) is an important parameter in the hydrodynamic modeling of oil fields. Shape of the curves of RPP physically is an expression heterogeneity of the porous medium (in particular, due to pore size distribution). In this paper we consider a simplified model for determining the RPP on the known functions of the pore size distribution, as well as the proposed model of capillary displacement of oil by water, which most accurately describes the results of laboratory experiments. The comparison of the RPP for a porous medium for different capillary models.*

Keywords: *relative permeability, capillary model, pore size distribution, reservoir simulation*

Oil development and forecast made on the basis of mathematical models of filtration of multiphase mixtures in porous media are described in many works [1, 5]. The parameters of a porous medium and fluids, usually determined by the results of numerous laboratory and field studies. One of the defining characteristics of multiphase filtration and at the same time the most uncertain and little-known are the RPP (the dependence of relative permeability from saturation phase).

There are different views about the nature of the two-phase (multiphase) flow in a porous medium, which may allow for the description of RPP. This may be mainly a piston displacement of one phase by other, or a joint flow in the pore channels, etc. The literature describes different approaches to determining the RPP [2, 6], is the basis for RPP are laboratory experiments, but because of their small number and inability to reach the desired volume and range of variation of reservoir properties, which characterizes the reservoir, then use various other methods of determining RPP [7], based on ideas about the mechanism of phase motion in a porous medium.

In this paper, based on the model of a porous medium as a set of capillary tubes of different diameters are illustrated features of RPP definitions and their relationship with microinhetereneity porous medium.

Consider the capillary model to determine the RPP described in the literature.

The first model: a hydrophilic capillary, through which water and oil flows, and water (as a wetting phase) flows along the capillary walls, and oil in the middle (Fig. 1) [9, 10].

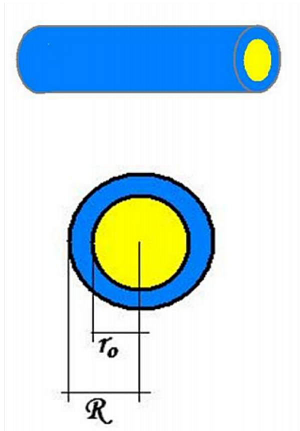

Fig. 1. The scheme of the model of interfacial interactions

OFP for the model are as follows:

$$k_1(S)=S^2\ , \quad (1)$$

$$k_2(S)=(1-S)\left(1-S\left(1-2\frac{\mu_1}{\mu_2}\right)\right), \quad (2)$$

where k_1, k_2 – RPP for water and oil; S – water saturation; μ_1, μ_2 – viscosity of water and oil, respectively, kg/(s·m).

In the second model it is assumed that the water flows on one capillaries (in a certain range of pore sizes), oil – on another (Fig. 2) [7].

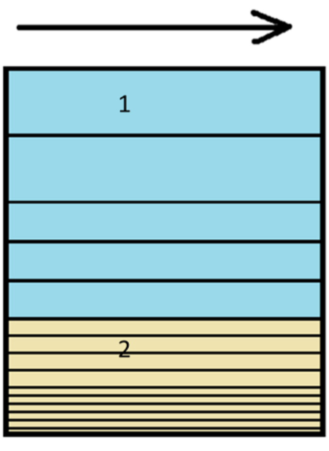

Fig. 2. The scheme of the second model

Permeability k, m^2; water saturation and RPP defined by the following formulas:

$$k=\frac{\int_0^\infty R^4 f(R)\,dR}{8\cdot\int_0^\infty R^2 f(R)\,dR}\ , \quad (3)$$

$$k_1(S)=k_1\left(\frac{\int_{R_w}^{\infty} R^2 f(R)dR}{\int_0^{\infty} R^2 f(R)dR}\right)=\frac{\int_{R_w}^{\infty} R^4 f(R)dR}{\int_0^{\infty} R^4 f(R)dR}, \tag{4}$$

$$k_2(S)=k_2\left(\frac{\int_{R_w}^{\infty} R^2 f(R)dR}{\int_0^{\infty} R^2 f(R)dR}\right)=\frac{\int_0^{R_w} R^4 f(R)dR}{\int_0^{\infty} R^4 f(R)dR}, \tag{5}$$

where R – radius of the capillary, m; $f(R)$ – function of pore size distribution, m^{-1}.

For the case of hydrophilic obtained similar results (changing limits of integration). You can see that in this model $k_1(S) + k_2(S) = 1$, therefore, are not independent.

The third model (percolation) is a two-dimensional grid of capillaries of different diameters [3]. The configuration of the grid is given by the dimension D, the number of connections converging on a single node z (coordination number), the percolation threshold r_c (Critical radius), m. Determining the parameters of the porous medium is determined by near and far from the percolation threshold $P_c = P(r_c)$.

The conductivity of the lattice:

$$\sigma=\begin{cases}\sigma_1,\ P(r_b)<P(r_1)\le 1,\\ \sigma_2,\ P(r_c)<P(r_1)\le P(r_b),\\ 0,\ 0\le P(r_1)\le P(r_c),\end{cases} \tag{6}$$

$$\sigma_1=\frac{1}{d^{(D-1)}}\frac{\pi r_m^4}{8}, \tag{7}$$

where d – the length of capillary, m; r_m – found from the equation, m:

$$\frac{1}{z/2-1}\int_0^{r_1} f(R)dR+\int_{r_1}^{\infty}\frac{r_m^4-R^4}{R^4+(z/2-1)r_m^4}f(R)dR=0, \tag{8}$$

$$\sigma_2=\frac{n(r_1)}{I(r_1)}, \tag{9}$$

$$I(r_1)=\frac{8}{\pi}\frac{\int_{r_1}^{\infty}\frac{f(R)}{R^4}dR}{\int_{r_1}^{\infty} f(R)dR}, \tag{10}$$

$$n(r_1)=\frac{\left(\int_{r_1}^{r_c} f(R)dR\right)^{v(D-1)}}{d^{(D-1)}}, \tag{11}$$

where v – an index of the correlation is a function of dimension D; r_c from the equation:

$$r_c=\int_{r_c}^{\infty} f(R)dR=\frac{D}{z(D-1)}. \tag{12}$$

Under the assumption that the medium is granular, and the pore size does not vary greatly, water saturation is equal probability $P(r_1)$.

$$P(r_1)=\int_{r_1}^{\infty} f(R)\,dR\,. \tag{13}$$

To find the permeability of oil, it is necessary in the formulas for calculating the conductivity change the limits of integration.

The models described above have certain drawbacks, chief among which is that none of them cannot reproduce the actual results of laboratory experiments (Fig. 3).

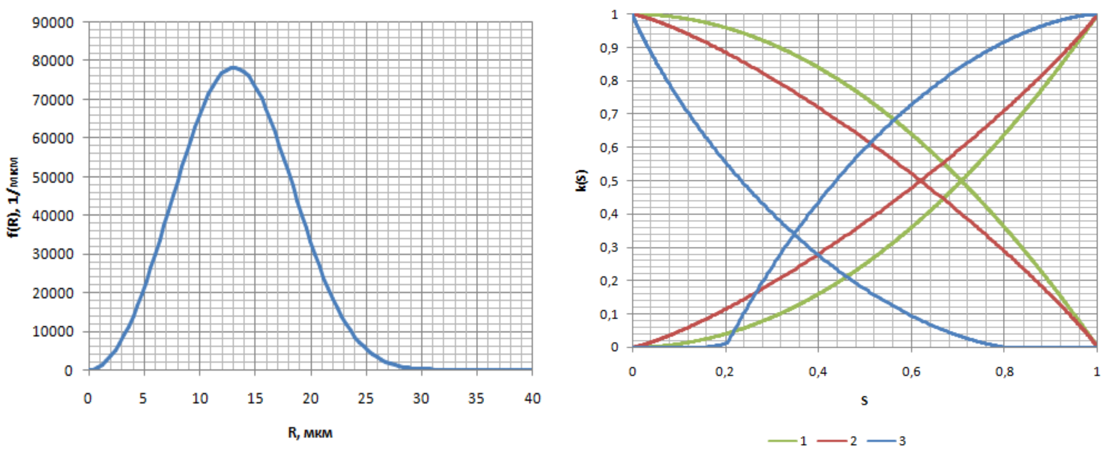

Fig. 3. Distribution function and the RPP received by him on the capillary model

In this paper we propose a slightly different model of capillary displacement of oil by water. The model represented by a set of capillaries with variable cross-section whose diameter is subject to some distribution law. The change in the diameter of each capillary is described by its distribution function, so that the hydraulic radius of the capillary together determine the general distribution function for the system. Take into account the capillary pressure and the nature of wettability of the porous medium. In the constrictions of the capillaries may be blocking the flow due to capillary forces (Fig. 4).

Initially, the law of motion is determined by the displacement front for a capillary of radius R (hydraulic radius R).

Obtain the solution:

$$p_x=\frac{p_1(L-x_f)+\frac{\mu_1}{\mu_2}p_2x_f}{L-x_f\left(1-\frac{\mu_1}{\mu_2}\right)}\,, \tag{14}$$

$$Q=\frac{\pi R^4}{8\mu_2}\frac{p_2-p_1}{L-x_f\left(1-\frac{\mu_1}{\mu_2}\right)}\,, \tag{15}$$

$$x_f = \frac{L}{1-\frac{\mu_1}{\mu_2}} \left(1 - \sqrt{1 - \frac{\left(1-\frac{\mu_1}{\mu_2}\right)(p_1-p_2) R^2 t}{4\mu_2 L^2}} \right), \tag{16}$$

where p_x – pressure on the border of the displacement, Pa; p_1, p_2 – pressure at the ends of the capillary, Pa; x_f – displacement front, m; L – length of the capillary, m; Q – flow rate in the capillary, m^3/s; t – time, s.

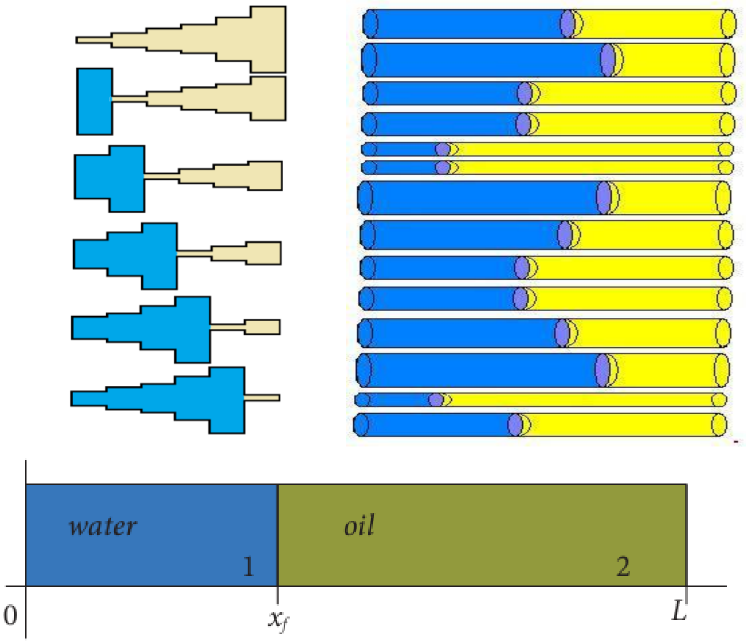

Fig. 4. The scheme of frontal displacement model

Breakthrough time of water in a capillary:

$$t_0 = \frac{4L^2(\mu_1+\mu_2)}{R^2(\Delta p + p_c)}, \tag{17}$$

where p_c – capillary pressure at the front of the displacement.

Water saturation for the model is determined by the following formula:

$$S = \frac{\int_0^\infty R^2 \left(\frac{x_f}{L}\theta(t,t_0) + 1 - \theta(t,t_0) \right) f(R) N_S(R,t)\,dR}{\int_0^\infty R^2 f(R)\,dR}, \tag{18}$$

where, $\theta = 1 - H$; H – Heaviside function; N – a function that is responsible for blocking channels.

Production rates of the system:

$$Q_1 = \int_0^\infty Q_1^1 (1-\theta(t,t_0)) f(R) N_1(R,t)\,dR, \tag{19}$$

$$Q_2 = \int_0^\infty Q_2^1 \theta(t,t_0) f(R) N_2(R,t)\,dR.$$

RPP found using Darcy's law:

$$Q_i = k \cdot \Omega \frac{k_i(S)}{\mu_i} \frac{dp}{dx}, \tag{21}$$

where i – phase (water or oil); Ω – cross-sectional area of the system, m^2.

The absolute permeability of the system is at full saturation of the i-th liquid:

$$k = \frac{Q_i \mu_i}{\Omega \frac{dp}{dx}}, \tag{22}$$

$$k_i(S) = \frac{Q_i(S)}{Q_i(S=1)}, \tag{23}$$

we obtain:

$$k_1(S) = \frac{\int_0^\infty R^4 f(R)(1-\theta(t,t_0)) N_1(R,t) dR}{\int_0^\infty R^4 f(R) dr}, \tag{24}$$

$$k_2(S) = \frac{\int_0^\infty R^4 f(R) \theta(t,t_0) \frac{L}{L - x_f\left(1 - \frac{\mu_1}{\mu_2}\right)} N_2(R,t) dR}{\int_0^\infty R^4 f(R) dr}. \tag{25}$$

Using the proposed model on a known distribution function we can determine the RPP (Fig. 5). At the same time in the presence of the experimentally obtained curves of RPP can be obtained form the distribution function of pore size, similar to the relationship of such distribution to the dependence of capillary pressure on saturation [8].

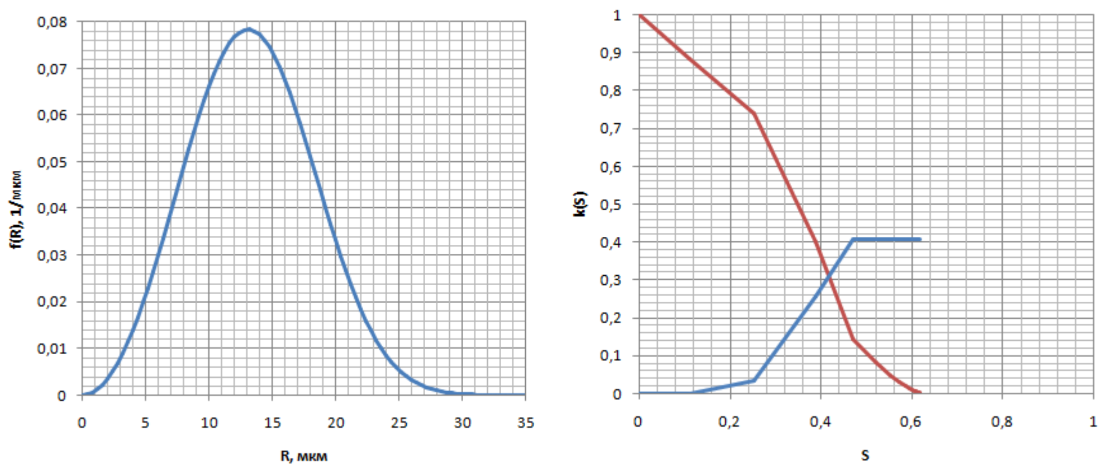

Fig. 5. RPP for the distribution function obtained by the proposed model

If there is a certain amount of RPP obtained by laboratory experiments on different samples of the porous medium, each of these experiments corresponds to its pore size distribution, so that in general, such distributions may differ. In this case, to describe the common features of the porous medium RPP should bear in mind that all these distributions are partial sample of the population and the total (or averaged) function corresponds to the RPP is the universe whose form should be obtained on the basis of statistical methods.

The functions of distribution may vary within wide limits, and depending on rock type (lithology) of the distribution function can be divided into two types: homogeneous and heterogeneous facies.For the homogeneous case of characteristic functions of the general distribution must comply with the same type of distribution which is observed in the partial samples. For non-homogeneous porous medium general population will be a weighted average of the share of different types of collectors (eg fractured porous medium) (Fig. 6, 7).

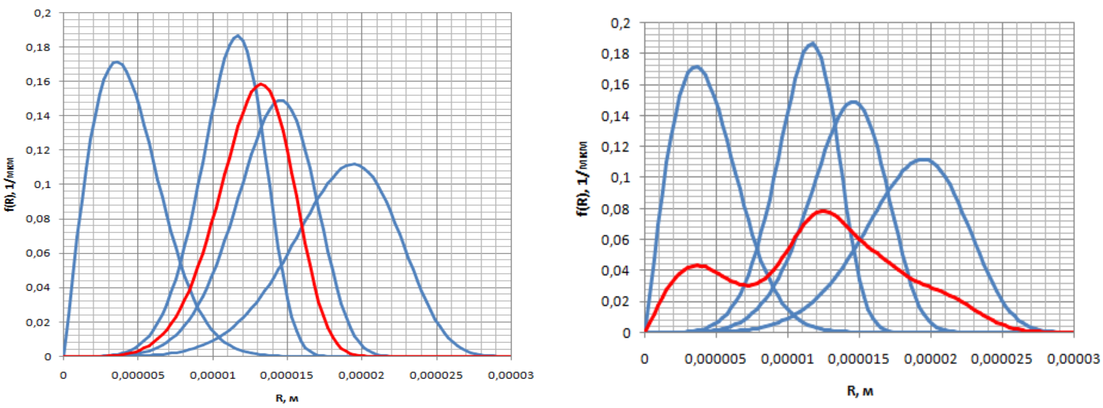

Fig. 6. Typical distribution functions for different types of rocks

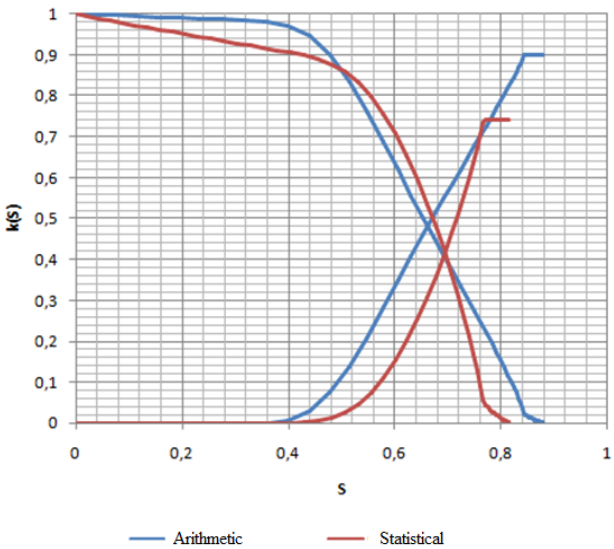

Fig. 7. RPP for statistical and arithmetic mean of the general distribution functions

Conclusions

In this paper we propose a model of capillary displacement of oil by water, which can be used to determine the RPP in the modeling of oil fields. Compared with the models proposed in the literature, the proposed model better reproduces the results of laboratory experiments. Since the RPP are characteristic of formation heterogeneity, in the presence of laboratory experiments to determine the RPP using the proposed model can be defined for them the function of pore size distribution and averaged to find the RPP for the reservoir as a whole.

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