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PRESSURE LOSS CALCULATION MODEL FOR WELL-GRADED SOLID-LIQUID PIPE FLOWS ON THE BASIS OF SYSTEMATIC PILOT PLANT INVESTIGATIONS

At designing pipeline transport of granulated solid materials, one of the most important parameter is the pressure loss over a given length of pipe. The pressure loss determines the applicable pump as well as the main portion of the operational costs. In the practice the transported solids can be characterized by size, density and shape distributions. Still many of the existing great number of pressure drop calculation methods describe mono-disperse solids. When solids are transported in horizontal pipelines by steady-state slurry flow, the most complicated case of interest of calculating the head loss is the flow of smaller flow rates (sliding bed flow may occur because operating the system close to the minimum point means low energy cost) of slurries with well granulated real solid material. The aim of this paper is to cope with this problem.

THE EFFECT OF PARTICLE SIZE

The most important parameter of a solid granular material system is the particle size in respect of the type of the two-phase flow. The effect of the particle size can be examined if all the other parameters are kept constant, only the size of mono-disperse solid fractions is changed and examined systematically. The Department of Process Engineering of the University of Miskolc, Hungary carried out two experimental series with power plant fly-ash and glass sand [8, 9, 10]. During the experiments narrow particle fractions were sieved and tested in a tube viscometer with three different pipes. The effect of the particle size can be summarized by the following figure:

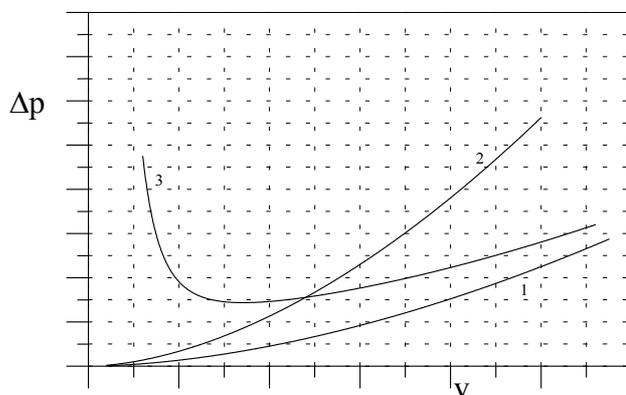


Fig. 1. The effect of the particle size.
(1 - water, 2 - fine-, 3 - coarse particles)

In Fig. 1 typical pressure drop curves are shown for two different particle sizes. The transport concentration and the pipe and all the other parameters are the same in both cases. In the case of small particles (2) the pressure loss curve is similar to the one of clear fluids. This kind of slurry flow was called as *fine suspension flow* [7]. In this case the solid-liquid mixture behaves like a clear fluid with its own density and rheological properties in the given pipe. This is like a continuum, and the

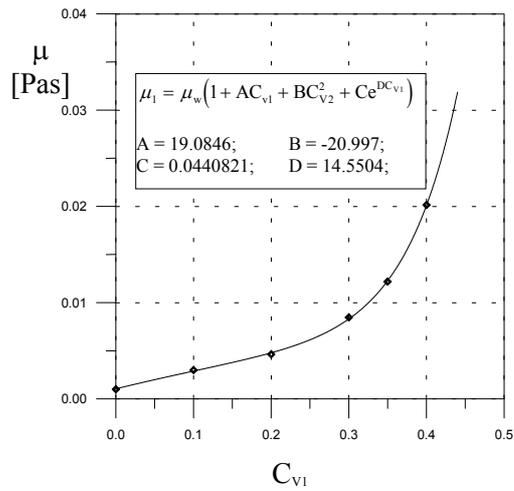


Fig. 2. The rheological properties of fine (5 - 30 μm) glass sand water suspensions (10)

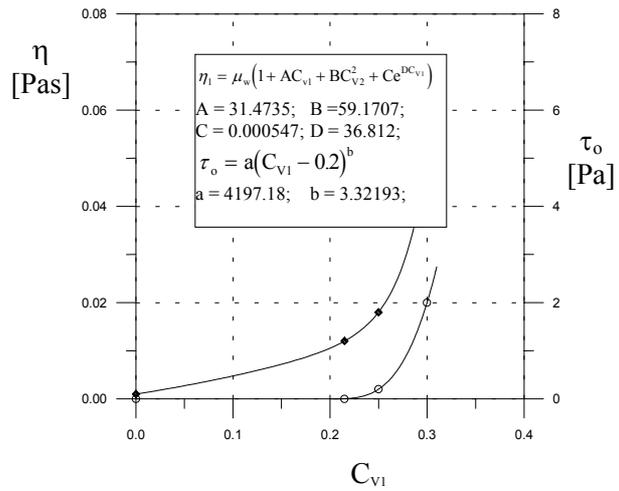


Fig. 3. The rheological properties of fine (<160 μm) fly-ash water suspensions (9)

continuum model can be well applied. The pressure loss can be calculated on the basis of the rheological properties of the suspension. Unfortunately the rheological properties of suspensions of fine powders and water are more non-Newtonian as the concentration increases. At this time general tensor level non-Newtonian constitutive equation does not exist but the artificial one dimensional rheological models such as Bingham-plastics, Exponential-Law, Yield-pseudoplastics, etc. can be well applied in the engineering practice. In a given practical case the rheological model and the parameters in it should be known as a function of the concentration. The pressure loss can be calculated by the methods of the literature for each rheological model.

The rheological properties of the examined glass-sand water suspension can be described by the Newtonian model up to 40% volumic transport concentration. In the case of the tested power plant fly-ash the rheological behavior changes at about 20%, a yield appears and the fluid becomes Bingham-plastics. In both cases the following function is good to describe the relation between the transport volumic concentration and the absolute viscosity of Newtonian or the rigidity coefficient of Bingham plastics fluids.

$$\mu_1 = \mu_w(1 + AC_{v1} + BC_{v1}^2 + Ce^{DC_{v1}}) . \quad (1)$$

The measured rheological properties of the glass sand - water from Fehérvárcsurgó in **Fig. 2** and the ones of the Mátra Power Plant fly-ash - water suspensions in **Fig. 3** are shown.

In the case of coarse particles (curve 3 in **Fig. 1**) the pressure loss curve has a minimum point. This case had been extremely wide examined in the literature and there is a fairly big misunderstanding in this respect as well. If the flow rate is sufficiently high the concentration distribution is symmetrical and this is called pseudo-homogeneous flow and sometimes rheological models are used for calculating the pressure loss. This is evidently wrong. In this case the continuum model is hard to be applied or even I think it cannot be applied. In this real two-phase flow the phenomenon

can be imagined as fluid flows between big rocks. This point of view is logically supported by the two-layer model [6], where the pressure loss of the sliding bed is calculated merely as mechanical friction between the pipe wall and the particles. What is surprising here: the transport of the same amount of coarse particles requires less energy than transporting fine particles in the same pipe. This is the case of experiments. In a given case the pressure loss curve should be measured and a function should be curve fitted to it. The first model to this type of flow was the Durand equation [1]. Evaluating our experiments [4, 9, 10] it was determined that the Durand equation was well applicable with changing the K multiplier and the n exponent as parameters in it for a given material. This kind of flow with coarse particles was called as *coarse mixture flow*. In an earlier paper a characterization method was proposed to distinguish the type of the flow by measuring of the pressure loss curve [7].

The transition as a function of the particle size from fine suspension flow into coarse mixture flow is not sharp. There is a transitional range where the exponent in the Durand equation starts to increase from 0 to the value of 3 in most of the cases. However, a transitional or critical particle size can be interpreted or measured by this way. Smaller particles than the critical size can be transported in the given pipe in a fine suspension flow.

The conclusion of these thoughts is that primarily the particle size determines the quality of the solid-liquid mixture flow. The actual flow patterns are then determined by the flow rate.

CALCULATION OF THE PRESSURE LOSS

Taking the above mentioned into account the following pressure loss calculation model can be proposed: The fine particles of a well graded solids system and the carrier fluid form a fine suspension. This is like the "vehicle" in the model of Wasp *et. al.* [5] except, that only the particle size determines this portion not the actual concentration distribution. The pressure loss of it is calculated by the rheological properties of this carrier fine suspension. The coarse particles are transported by this fine suspension. Unfortunately, because the transition particle size range the graded solids should be characterized by discrete particle fractions. It means that the pressure loss can be a sum of the head loss caused by each coarse particle fraction. If the flow rate is sufficient the coarse particles are suspended and the additional heterogeneous pressure loss can be correlated by the modified Durand equation. If there is a sliding bed the head loss of the settled particles can be calculated as mechanical friction between them and the pipe wall in accordance with the two-layer model.

Division of the solids content

Let's introduce a notation for the division of the transported solid content. There are 'N' particle fractions, 'i' is the index of the actual fraction. Multiplying the volumic transport concentration ' C_V ' by the percentage of each particle fraction results the volumic concentration of each fraction ' C_{Vi} '

$$C_{Vi} = C_V X_i, \quad (2)$$

where, X_i is the percentage of particle size fractions, obtained for example by sieving. In addition the density ' ρ_{Si} ' and the shape coefficient ' SF_i ' of each fraction characterize

the granulated solid material. The finer the resolution of the separation of particle fractions, the better the characterization of solids is. According to the size of the fractions the solid content can be divided into two parts: the smaller particles than the critical size form a stable fine suspension with the clear carrier liquid, this is signed by the '1' subscript. The coarser particles can be transported in heterogeneous mixture flow. It is signed by the '2' subscript.

$$\begin{aligned}
 C_{v1i} &= C_{vi} & \text{for } d_i < d_{limit} ; \\
 C_{v2i} &= C_{vi} & \text{for } d_i \geq d_{limit} .
 \end{aligned}
 \tag{3}$$

The concentration of the fine suspension part ' C_{v1} ' can be calculated by summing up the fraction concentrations of which particle size is smaller than the transitional limit particle size. Summing up the volumetric fraction concentrations of coarser gives the volumetric concentration of coarse particles:

$$C_{v2} = \sum_{i=j}^N C_{v2i} \quad \text{and} \quad C_{v2i} = C_{v2upi} + C_{v2loi} ,
 \tag{4}$$

where, 'j' is the index of the first particle fraction of which particle size is coarser or equal to the limit. The fraction percentages have to be written in ascending order of the particle size. Fraction concentrations of coarse particles can be divided into two parts according to the current position of the particles. ' C_{v2upi} ' is the fraction volumetric concentration of the i-th coarse particle fraction to be suspended. ' C_{v2loi} ' is the fraction volumetric concentration of the i-th coarse particle fraction to be in the sliding bed. The '1' and '2' indexes refer to fines and coarser. The 'up' and 'lo' indexes refer as suspended load and bed load of the coarser. By the following equation these concentrations can be defined:

$$C_v = \sum_{i=1}^{j-1} C_{v1i} + \sum_{i=j}^N C_{v2upi} + \sum_{i=j}^N C_{v2loi} .
 \tag{5}$$

For applying and developing the model a criterion is necessary to determine these concentrations. The Wasp *et. al* [5] method applies the Ismail equation to correlate the concentration at 0.08D of the top of pipe. This method works very well for fully suspended flow only. Shook *et. al* [2] proposed an experimental equation. The disadvantage of the model that it predicts all the solids to be in the sliding bed if the flow velocity is zero, and all solids are suspended if the flow velocity tends to infinity. Let ' v_{m2} ' mean the velocity at which the first particles start to slide in the bottom of the pipe. With other words, at higher velocities than ' v_{m2} ' the actual concentration at the bottom of the pipe is smaller than the free settled concentration, all the solids are suspended. With decreasing flow rate at ' v_{m2} ' a sliding layer starts to form of which concentration is close to the free settled concentration. With more flow rate decreasing, at the velocity ' v_{m3} ', all the coarser are in the bed. The two end points of the criterion are known. The following function can describe this situation.

$$\left(\frac{C_{V2up}}{C_{V2}}\right) = \exp\left[-K\left(\frac{1}{v - v_{m3}} - \frac{1}{v_{m3} - v_{m2}}\right)\right]. \quad (7)$$

The 'K' constant needs more consideration. The criterion may be searched in this form. Unfortunately, at this time there are no measured data at the author's disposal to establish an experimental equation. Looking at the other models, and after consideration the criterion seems to depend upon the settling behavior of particles, the actual rate of shear, the relative magnitude of the pipe comparing to the particles and the rate of hindered settling (volumic concentration). The settling behavior of particles are taken into account by the terminal settling velocity in Ismail's equation, and by the Archimedes number in Shook's equation. The terminal settling velocity represents more information. The actual rate of shear is taken into account by the average flow velocity. The pipe diameter - particle size ratio is taken into account in Shook's equation as $(D/d)^{0.431}$. It's exponent is 0.431. At higher concentrations, particles disturb each other causing less sedimentation. This effect of hindered settling may be taken into account by the volumic transport concentration - free settled volumic concentration ratio. The criterion might be the following for each particle fraction of which size is bigger then the limit particle size.

$$\left(\frac{C_{V2upi}}{C_{V2i}}\right) = 1, \quad \text{if } v > v_{m2}; \quad (8)$$

$$\left(\frac{C_{V2upi}}{C_{V2i}}\right) = \exp\left\{-v_{ti}\left(\frac{D}{d_i}\right)^{0.431}\left(\frac{C_v}{C_{vmax}}\right)^{0.5}\left[\frac{1}{v - v_{m3}} - \frac{1}{v_{m3} - v_{m2}}\right]\right\}, \quad \text{if } v_{m3} < v \leq v_{m2}; \quad (9)$$

$$\left(\frac{C_{V2upi}}{C_{V2i}}\right) = 0, \quad \text{if } v \leq v_{m3}. \quad (10)$$

These equations have not been confirmed by experiments yet.

Calculation of the pressure drop

Streat [3] simplified the two-layer model as that the total pressure loss of a sliding bed slurry flow can be divided into two parts: -frictional energy loss due to the suspended solids - liquid mixture in the whole perimeter of pipe and -energy loss due to mechanical sliding between the moving bed and the pipe wall.

$$\Delta p = \Delta p_{susp} + \Delta p_{bed}, \quad (11)$$

' Δp_{susp} ' may be divided into two parts in accordance with the earlier described point of view: pressure loss of the fine suspension portion and overpressure due to the suspended coarse particles:

$$\Delta p = \Delta p_1 + \Delta p_{2up} + \Delta p_{bed}. \quad (12)$$

Calculation of the pressure loss of the fine suspension portion

The fine suspension part of the slurry flow (the clear carrier liquid and the fines) contains the fine particles in ' $C_{V1}/(1-C_V+C_{V1})$ ' concentration. The pressure loss of this part ' Δp_1 ' can be calculated on the basis of the rheological behavior of this suspension. In a given case the rheological parameters have to be known as a function of the concentration of fines for a given temperature. It may require pilot plant measurements. The following one dimensional rheological models might be applied in the engineering practice:

- Newtonian $\tau = \mu \left(-\frac{du}{dr} \right);$
- Bingham-plastic $\tau = \tau_o + \eta \left(-\frac{du}{dr} \right);$
- Power Law $\tau = K \left(-\frac{du}{dr} \right)^m ;$
- Yield-pseudoplastic $\tau = \tau_o + K \left(\frac{du}{dr} \right)^m .$

The pressure loss of a fine suspension can be calculated by the Darcy and Weisbach equation. In the literature there are several works on how to calculate the Fanning friction factor on the basis of the rheological behavior. In this paper only the names of the models and equations are mentioned for this task. They have been selected in an extensive computer comparison work carried out earlier by the author [10]. In the case of Newtonian fluids the Colebrook equation is applicable in the turbulent field for any pipe roughness. For Bingham plastic fluids the Buckingham equation in the laminar field, the method of Hanks in the turbulent field and for the estimation of the laminar - turbulent transition are the suggested models. In the case of Power Law fluids the usage of the so called Power Law Reynolds number is recommended. By this way the Fanning friction factor can be calculated similarly to the Newtonian fluids in the laminar field. In the turbulent field where the pressure loss depends on the pipe roughness the BNS equation; for correlating the laminar - turbulent transition the Ryan and Johnson equation are applicable. For the Yield-Pseudoplastic fluids the method of Hanks is suggested.

Calculation of the energy loss due to the coarse particles

For the calculation of the heterogeneous overpressure due to the suspended coarse particles, there are so many equations in the literature. They are the so called Durand type equations. Tarján and Debreczeni [4] concluded that the reason of the significant scatter among them is due to they are experimental equations and the quality of the mixture flow was in the transient fine suspension - coarse mixture zone during many of the experiments. The Durand equation can be generalized by treating the constant 'K' and the exponent of the Froude number 'n' in it as material dependent parameters. 'K₁' and 'n₁' have to be measured for discrete particle fractions in the given pipe with the given material. The 'n' parameter is suitable to characterize the quality of the flow. The heterogeneous overpressure due to the suspended coarse particles ' Δp_{2up} ' may be calculated by the modified Durand equation.

$$\Delta p_{2upi} = \Delta p_1 K_i C_{v2upi} \frac{\left(\frac{\rho_{si}}{\rho_1} - 1\right)^{1.5}}{C_{Di}^{0.75}} \frac{1}{\left(\frac{v}{\sqrt{gD}}\right)^{n_i}} \quad (13)$$

for each coarse fraction, and the total is

$$\Delta p_{2up} = \sum_{i=j}^N \Delta p_{2upi} \quad (14)$$

The drag coefficient is calculated on the basis of the literature. Coarse particles of the given fraction are settling in the fine suspension part, in the vehicle.

The pressure loss due to mechanical sliding may be calculated as Streat *at. al* proposed:

$$\Delta p_{bed} = \frac{2}{\pi} \rho_m g (\sin \beta - \beta \cos \beta) \mu_s \left(\frac{\rho_s}{\rho_1} - 1\right) C_{vmax} \quad (15)$$

The angular position of sliding bed ' β ' can be calculated if the ' C_{v2lo} ' concentration fraction is known. ' C_{v2lo} ' may be calculated from Equations (2 - 10). The limit velocity ' v_{m2} ' of sliding bed forming may be calculated as the minimum point of the pressure loss curve calculated by the assumption of no any particles in the sliding bed (by the modified Durand equation). ' v_{m3} ' can be assumed to be zero as a rough estimation. The cross sectional area occupied by the coarse particles in the bed can be expressed as:

$$A_{lo} = A \frac{C_{v2lo}}{C_{vmax}} \quad (16)$$

' β ' is calculated by iteration from

$$A_{lo} = \frac{D^2 (\beta - \sin \beta \cos \beta)}{4} \quad (17)$$

The model is not iterative. It can be recommended for application of any kind of mixture flow with flow rates higher then ' v_{m3} '.

DETERMINATION OF THE MATERIAL PARAMETERS

For the application of the model for pipeline transport design non-usual parameters have to be known. These are: the critical or limit particle size ' d_{limit} ' between the fine suspension - coarse mixture flow, the ' K_i ' and ' n_i ' parameters for the coarse particle fractions, the mechanical friction factor between the pipe wall and the solid particles ' μ_s ' and the rheological properties of the fine fractions and the clear carrier liquid formed fine suspensions as functions of the concentration and the temperature. These parameters can be estimated on the basis of the literature or they can be measured.

For carrying out such measurements the taken sample from the transported solid material has to be prepared, namely it has to be separated into particle fractions. The pressure loss curves of the prepared mixtures made from each fraction and the clear liquid in different concentrations have to be measured by tube viscometer or by some kind of hydraulic loop. The diameter of the measuring pipe of the test loop should be as close to the designed one as possible, since the pipe diameter has an important role to the quality of the flow. Even very fine particles can be transported in a tiny capillary tube as rough heterogeneous mixture.

The first step of the evaluation process is the calculation of the left side of the next equation.

$$\frac{\frac{\Delta p}{\Delta p_w} - 1}{C_v \left(\frac{\rho_s}{\rho_w} - 1 \right) \frac{1}{C_D^{0.75}}} = K_i \frac{1}{Fr^{n_i}} \quad (18)$$

In the cases of the measuring series, the transport concentration ' C_v ', the density of the solid fraction ' ρ_s ' and the drag coefficient ' C_D ' are constants. The left side of the equation is proportional to the pressure loss caused by the presence of the solid material in the function of the average flow velocity. This equation can be written as

$$\phi = K_i \left(\sqrt{gD} \right)^{n_i} v^{-n_i} \quad (19)$$

by signed by ' ϕ ' the left side and by substituting the Froude number. By this way for the given pipe, ' $\phi - v$ ' value pairs are at the designer's disposal. ' K_i ' and ' n_i ' can be determined by fitting Equation (19) for each particle fraction. According to Tarján and Debreczeni, where $n_i < 0.5$ the fractions are fines, where $n_i > 0.5$ the fractions are coarses. By this way the fine suspension - coarse mixture transitional particle size ' d_{limit} ' can be determined as well. In the case of the fine fractions, the rheological behavior of these suspensions has to be measured. The mechanical friction factor between the pipe wall and the particles can be determined by the tilting tube method.

PRACTICAL USE AND AN EXPERIMENTAL PROOF OF THE MODEL

If the particle size distribution of the transported solid material is such where discrete fractions can be well distinguished, fines, coarses and medium sizes as well, the division of the solid system into many fractions may be necessary. However, this means lots of calculations and measurements. The model can be simplified if the solid system is divided into only two parts: fines and coarses by the help of the transitional particle size. By this way a very powerful and fairly simple model is at our disposal.

In 1998 our department carried out some experiment with fly-ash and slag - water mixtures in a hydraulic loop as being commissioned by the Power Plant of Mátra. From our earlier experiments it was known that the transitional limit particle size of fly-ash is about 160 μm . Therefore, we separated at this size some part of the supplied sample and measured the rheological properties as a function of the concentration of the fine portion. The result is shown in **Fig. 3**. Afterwards, the complex sample material was

tested in a hydraulic loop containing 53, 75, and 120 mm pipes. The measured pressure loss values of a representative case are shown in **Fig. 4** labeled by small triangles. The volumic transport concentration was 33.8%.

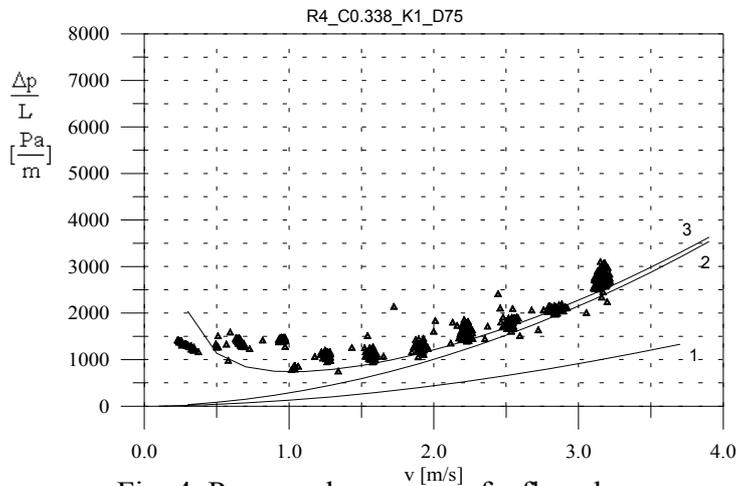


Fig. 4. Pressure loss curve of a fly-ash, slag and water mixture

The calculated and checked by measurements pressure loss curve of the clear carrier water is shown in **Fig. 4**, signed by 1. Because the particle size distribution was measured of the tested sample material, the portion of fine particles ($< 160 \mu\text{m}$) was determined. Using this ratio the concentration of the fine suspension portion was calculated and the rheological properties and the parameters were obtained by **Fig. 3**. The

pressure loss curve (2) in **Fig. 4** was calculated afterwards by the referred models described earlier. The proof of the proposed model means, that the measured pressure loss values take place closer and closer to curve (2), the pressure loss curve of the fine suspension portion with increasing flow velocity instead of the one of water (curve 1). This is very well known, that the pressure loss of a coarse mono-disperse mixture approaches the pressure loss of water for high flow rates. The predicted pressure loss curve by the proposed model is shown as well in **Fig. 4** (curve 3).

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SYMBOLS USED

A	- cross-sectional area	m	- exponent of Power Law fluids
C_D	- drag coefficient	N	- number of particle fractions
C_V	- transport volumic concentration	SF	- shape coefficient
C_{Vmax}	- free settled solids concentration	v	- average mixture velocity
d, x	- particle size	v_{m2}	- deposition limit velocity
d_{limit}	- transitional particle size	v_{m3}	- at this velocity all the coarse solids are in the bed
D	- pipe diameter	v_t	- terminal settling velocity
Fr	- Froude number	X	- particle fraction percentage
g	- acceleration due to gravity	β	- angular position of the bed
K	- parameter in the modified Durand equation	η	- coefficient of rigidity
K	- consistency index	ρ	- density
n	- parameter in the modified Durand equation	μ	- absolute viscosity

μ_S	- mechanical friction coefficient	j	- index of the fraction belongs to the limit particle size
Δp	- specific pressure loss	1	- fine fractions
τ	- shear stress	2	- coarse fractions
τ_o	- yield stress	up	- suspended
$(-du/dr)$	- shear rate	lo	- bed load
		S	- solid material
		V	- volumic
		W	- carrier liquid (water)
		m	- mixture
i	- i-th particle fraction		

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