Mihály Dobróka

Geophysical Department,
University of Miskolc, Hungary

ROCK STRESS DETERMINATION IN BOREHOLES WITHOUT OVERCORING

The paper deals with the simultaneous determination of five independent stress components from the data measured in a single borehole. The proposed technique consists of cementing a solid cylindrical epoxy probe in which strain gauges have been embedded into a hole predrilled into the rock mass. Using the rheological equations of the Poynting-Thomson body analytical solutions are given for the displacement and strain field of the probe and of the rock. By measuring the time history of the deformations of the probe one can determine stress components occurring in the rock continuum without overcoring of the borehole.

1. INTRODUCTION

The stress-relief technique of in situ stress measurements by overcoring is well known. The first method of this type developed by Leeman (1968) uses rosette strain gauges directly cemented on the surface of the borehole. In order to overcome the strain gauge waterproofing problems occurring in wet conditions Rocha and Silverio (1969) developed the solid inclusion technique, using strain gauges embedded in a cylindrical epoxy probe. A similar «soft-inclusion» device with an elastic modulus less than that of the host material was also utilized by Blackwood (1977). In order to reduce the bond stresses occurring at the interface between the rock and the probe a new technique (the so-called hollow inclusion) was developed by Rocha et al. (1974) and Worotnicki and Walton (1976) using a tubular probe with embedded gauges. The complete displacement and stress field of the probe and that of the rock mass were given by Savin (1951), Duncan-Fama and Pender (1980).

As a material equation, in all the works mentioned Hooke's law was used. This results in the necessity for overcoring. The steps of the rock stress measurements are shown in **Fig. 1**. In consequence of the overcoring the rock core expands together with the cemented probe, producing tensile stresses at the epoxy-rock interface, which may be sufficently large to break the bond between the rock and the epoxy probe. This is the case even for the hollow probe, when the rock material is «soft» (Duncan-Fama and Pender 1980). There is also another disadvantage in connection with overcoring: it can cause cracks and failures in the investigated rock annulus resulting in erroneous measurements.

In what follows we propose a new method based on the rheological equation of the Poynting-Thomson body. The rheological features of the rock continuum result in time dependent displacements and strains in it after the borehole has been drilled. On measuring these straings at various times, the in situ rock stresses can be determined without overcoring the borehole.

2. OPERATION PRINCIPLES

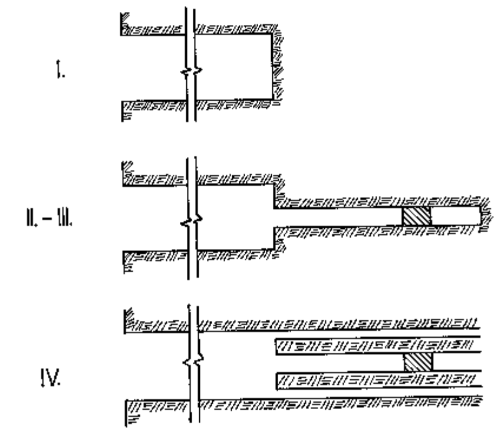

Fig. 1. Steps of measurements
I – Drilling a borehole to the neighbourhood of the point, where the stress is to be determined; II – Drilling a coaxial hole with a smaller diameter; III – Cementing the probe with embedded strain gauges into the pilot hole; IV – Overcoring the rock and observing the strain induced in the instrument

As is well known, in many cases Hooke's law gives only an approximate description of the real rock continua. In a wide range of phenomena the rocks may have time-dependent properties. In order to take the observed time-dependence into account (Kurlenya et al. 1973) used time-dependent moduli in the computation of the deformations of the borehole wall. The same method was applied by Senuk (1973) who calculated the stresses appearing in a photo-elastic probe cemented into the borehole.

There are some rock properties (strain retardation and stress relaxation, etc.) which require that the time-derivatives of the material equation be incorporated. The most simple rheological equation describing relaxational and retardational processes is the Poynting-Thomson model (standard body) where the relationship between the stress and deformation deviators T_{ik} and E_{ik} is given as a differential equation

$$\left\{1+\tau\frac{\partial}{\partial t}\right\}T_{ik}=\left\{1+\vartheta\frac{\partial}{\partial t}\right\}2GE_{ik} \tag{1}$$

while the spherical part of the stress and of the deformation tensors $T_{ik}^{(0)}$ and $E_{ik}^{(0)}$ satisfies a linear equation

$$T_{ik}^{(0)}=3KE_{ik}^{(0)} \tag{2}$$

where G is the shear modulus, τ the relaxation time, ϑ the retardation time, K the modulus of compression. In slow processes, when $\vartheta << T$ (T being the characteristic time of the process) Eqs. **1**, **2** can be written as

$T_{ik}=2GE_{ik}$ and $T_{ik}^{(0)}=3KE_{ik}^{(0)}$

while in quick processes, when $\tau >> T$

$T_{ik}=2G*E_{ik}$ and $T_{ik}^{(0)}=3KE_{ik}^{(0)}$

where $G^* = G\frac{\vartheta}{\tau}$ is the dynamic shear modulus. In these two limiting cases the Poynting-Thomson body can be treated as a linearly elastic one. In the model the relation $\vartheta \geq \tau$ is fulfilled, so in our considerations $G^* \geq G$.
Since the values of the rheological times, ϑ, τ are of the order of some ten hours, the drilling of a borehole can be regarded as a quick process. During the drilling process, strains occur $E_{ik}(0) = \frac{1}{2G^*}T_{ik}$ in the host rock, which cannot be measured directly.
Using Leeman's method, or cementing a soft hollow probe into the bore-hole, the stress state of the rock mass can be approximately considered as independent of time: $\dot{T}_{ik} = 0$ (the dot denoted differentiation with respect to the time). By Eq. 1 the strains can be written as

$$E_{ik} = \frac{1}{2G}T_{ik}\left[1 - \left(1 - \frac{\tau}{\vartheta}\right)e^{-t/\vartheta}\right]. \tag{3}$$

The measurable quantity

$$E_{ik} - E_{ik}(0) = \frac{1}{2G}T_{ik}\left((1 - \frac{\tau}{\vartheta}\right)(1 - e^{-t/\vartheta})$$

depends on the in situ stresses and – through its time dependence – gives a possibility to determine the rheological parameter ϑ.
It will be shown later that in real situations, when an elastic probe is used, we can measure four independent time parameters characteristic of the interaction between the probe and the rock media, and five independent strains of the probe. By means of these quantities we can determine five independent elements of the in situ stress tensor and the material parameters *G*, *K*, ϑ, τ as well.

3. ANALYTICAL SOLUTION FOR THE DISPLACEMENT FIELDS

The analytical solution for displacements and strains is required in order to relate the strains measured in the probe to the stresses occuring in the rock. The results referring to the linear elasticity are also given in Savin (1951) and Duncan-Fama et al. (1980). The solutions for the case when the rock material has rheological properties determined by the Poynting-Thomson model can differ only in their time dependence.
The in-situ rock stresses can be written in cylindrical coordinate system, as

$$\sigma_r^{(0)} = \frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)}) + \frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)})\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta;$$

$$\sigma_\theta^{(0)} = \frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)}) + \frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)})\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta;$$

$$\tau_{r\theta}^{(0)} = -\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)})\sin 2\theta + \tau_{xy}^{(0)}\sin 2\theta;$$

$$\tau_{rz}^{(0)} = \tau_{yz}^{(0)}\sin\theta + \tau_{xz}^{(0)}\cos\theta;$$

$\tau_{\theta z}^{(0)} = \tau_{yz}^{(0)} \cos\theta + \tau_{xz}^{(0)} \sin\theta$;

$\sigma_z^{(0)} = \sigma_z^{(0)}$,

where $\sigma_x^{(0)}, \sigma_y^{(0)}, \sigma_z^{(0)}, \tau_{xy}^{(0)}, \tau_{xy}^{(0)}, \tau_{yz}^{(0)}$ are the stresses in a Cartesian coordinate system. While drilling a borehole and cementing a probe to it, there occur changes in the stress state determined by the equilibrium equations

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0;$$

$$\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} = 0;$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\tau_{rz}}{r} = 0.,$$

where $\sigma_r, \sigma_\theta, \tau_{r\theta}, \tau_{rz}, \tau_{\theta z}$ are total stresses in the rock. The material equations can be written in the form

$$\sigma_r^* + \tau\dot{\sigma}_r^* = 2G\left\{e_r + \frac{\nu\theta}{1-2\nu} + \vartheta\left(\dot{e}_r + \frac{\nu\mu\dot{\theta}}{1-2\nu}\right)\right\};$$

$$\sigma_\theta^* + \tau\dot{\sigma}_\theta^* = 2G\left\{e_\theta + \frac{\nu\theta}{1-2\nu} + \vartheta\left(\dot{e}_\theta + \frac{\nu\mu\dot{\theta}}{1-2\nu}\right)\right\};$$

$$\sigma_z^* + \tau\dot{\sigma}_z^* = 2G\left\{e_z + \frac{\nu\theta}{1-2\nu} + \vartheta\left(\dot{e}_z + \frac{\nu\mu\dot{\theta}}{1-2\nu}\right)\right\};$$

$$\tau_{r\theta}^* + \tau\dot{\tau}_{r\theta}^* = 2G\{e_{r\theta} + \vartheta\dot{e}_{r\theta}\};$$

$$\tau_{rz}^* + \tau\dot{\tau}_{rz}^* = 2G\{e_{rz} + \vartheta\dot{e}_{rz}\};$$

$$\tau_{\theta z}^* + \tau\dot{\tau}_{\theta z}^* = 2G\{e_{\theta z} + \vartheta\dot{e}_{\theta z}\},$$

where $\sigma_{r,}^* \, \sigma_\theta^*, \sigma_z^*, \tau_{r\theta}^*, \tau_{rz}^*, \tau_{\theta z}^*$ are stresses appearing in consequence of the borehole opening, ν is Poisson's ratio,

$$e_r = \frac{\partial u}{\partial r}, \qquad e_\theta = \frac{1}{r}\left(u + \frac{\partial \upsilon}{\partial \theta}\right), \qquad e_z = \frac{\partial w}{\partial z};$$

$$e_{r\theta} = \frac{1}{2r}\left(\frac{\partial u}{\partial \theta} - \upsilon + r\frac{\partial \upsilon}{\partial r}\right), \qquad e_{\theta z} = \frac{1}{2}\frac{\partial w}{\partial \theta};$$

$$e_{rz} = \frac{1}{2}\frac{\partial w}{\partial r}, \qquad \theta = e_r + e_\theta + e_z$$

and $u(r,\theta,t), \upsilon(r,\theta,t), w(r,\theta,t)$ are the displacements.
The time of the drilling of the borehole is very small in comparison with the rheological time parameters, so while drilling, the rock can be regarded as a linearly elastic continuum with the shear modulus G^*. So the displacements of the rock are

$$u^* = \frac{R}{2G^*}\left\{\frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)})\frac{R}{r} + \left[(\chi^* + 1)\frac{R}{r} - \left(\frac{R}{r}\right)^3\right]\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}\cos 2\theta + \tau_{xy}\sin 2\theta\right]\right\};$$

$$\upsilon^* = \frac{R}{2G^*}\left\{(\chi^* - 1)\frac{R}{r} + \left(\frac{R}{r}\right)^3\right\}\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right];$$

$$w^* = \frac{R}{2G^*}\left\{\tau_{xz}^{(0)}\cos\theta + \tau_{yz}^{(0)}\sin\theta\right\}\frac{R}{r}, \qquad \chi^* = 3 - 4v^*.$$

Similarly assuming that the setting time of the cement for fixing the probe into the borehole is sufficiently smaller than the relaxation time τ, the initial conditions can be taken as

$$u^{II}(0) = u^* \qquad \upsilon^{II}(0) = \upsilon^* \qquad w^{II}(0) = w^*$$

for the rock and

$$u^I(0) = u(0) \qquad \upsilon^I(0) = \upsilon(0) \qquad w^I(0) = w(0)$$

for the probe at $t = 0$. The boundary conditions – neglecting the deformations of the thin cement layer between the probe and borehole surface – can be written in the form

$$u^I = u^{II} - u^* \qquad \upsilon^I = \upsilon^{II} - \upsilon^* \qquad w^I = w^{II} - w^*;$$

$$\sigma_r^I = \sigma_r^{II} \qquad \tau_{r\theta}^I = \tau_{r\theta}^{II} \qquad \tau_{rz}^I = \tau_{rz}^{II}$$

at $r = R$, R being the radius of the borehole and

$$\sigma_r^{II} = \sigma_r^{(0)} + \sigma_r^* \qquad \tau_{r\theta}^{II} = \tau_{r\theta}^{(0)} + \sigma_{r\theta}^* \qquad \tau_{rz}^{II} = \tau_{rz}^{(0)} + \tau_{rz}^*;$$

or using the material equations one finds

$$\sigma_r^{(0)} + 2G\left\{e_r^{II} + \frac{v\theta^{II}}{1-2v} + \vartheta\left[\dot{e}_r^{II} + \frac{v\theta^{II}}{1-2v}\right]\right\} = 2G_1\left\{e_r^I + \frac{v\theta^I}{1-2v_1} + \tau\left(\dot{e}_r^I + \frac{v_1\theta^I}{1-2v_1}\right)\right\};$$

$$\tau_{r\theta}^{(0)} + 2G\left\{e_{r\theta}^{II} + \vartheta \dot{e}_{r\theta}^{II}\right\} = 2G_1\left\{e_{r\theta}^{I} + \tau \dot{e}_{r\theta}^{I}\right\};$$

$$\tau_{rz}^{(0)} + 2G\left\{e_{rz}^{II} + \vartheta \dot{e}_{rz}^{II}\right\} = 2G_1\left\{e_{rz}^{I} + \tau \dot{e}_{rz}^{I}\right\},$$

where G_1, v_1 are the elastic parameters of the probe. We seek the solution in a form similar to that in the Hookean case:

$$u^I = \frac{R}{4G_1}\left\{(\chi_1 - 1)a\frac{r}{R}\frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)}) + \left[b\frac{r}{R} + (\chi_1 - 3)c\left(\frac{r}{R}\right)^3\right]\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right]\right\}$$

$$\upsilon^I = \frac{R}{4G_1}\left\{(\chi_1 + 3)c\left(\frac{r}{R}\right)^3 - b\frac{r}{R}\right\}\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right];$$

$$w^I = \frac{R}{G_1}d\frac{r}{R}\left[\tau_{xz}^{(0)}\cos\theta + \tau_{yz}^{(0)}\sin\theta\right];$$

$$u^{II} = -\frac{R}{2G}\left\{A\frac{r}{R}\frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)}) - \left[(\chi + 1)B\frac{R}{r} + C\left(\frac{R}{r}\right)^3\right]\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right]\right\}$$

$$\upsilon^{II} = \frac{R}{2G}\left\{(\chi - 1)B\frac{R}{r} - C\left(\frac{R}{r}\right)^3\right\}\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right];$$

$$w^{II} = \frac{R}{G}D\frac{R}{r}\left[\tau_{xz}^{(0)}\cos\theta + \tau_{yz}^{(0)}\sin\theta\right],$$

where a, b, c, d, A, B, C, D fulfil the set of equations

$$1 + A + \vartheta\dot{A} = a + \tau\dot{a};$$

$$1 - \left\{4B + 3C + \vartheta\left[4\dot{B}(1-\kappa) + 3\dot{C}\right]\right\} = \frac{b}{2} + \tau\frac{\dot{b}}{2};$$

$$1 + 2B + 3C + \vartheta(2\dot{B} + 3\dot{C}) = \frac{b}{2} - 3c + \tau\left(\frac{\dot{b}}{2} - 3\dot{c}\right);$$

$$1 - D - \vartheta\dot{D} = d + \tau\dot{d};$$

$$(\chi_1 - 1)a = -2\varepsilon\left(A + \frac{G}{G^*}\right);$$

$$b+(\chi_1-3)c=2\varepsilon\left[(\chi+1)B+C-\chi^*\frac{G}{G^*}\right];$$

$$(\chi_1+3)c-b=-2\varepsilon\left[(\chi-1)B-C-\chi^*\frac{G}{G^*}\right];$$

$$d=\varepsilon\left(D-\frac{G}{G^*}\right)\ \text{with}\ \chi_1=3-4\nu_1,\ \varepsilon=\frac{G_1}{G},\ \kappa=\frac{1}{3}(1+\nu)\left(1-\frac{\tau}{\vartheta}\right).$$

Taking into account the initial conditions we find

$$A=\frac{\vartheta_1}{\vartheta}\left[-1+\left(1-\frac{\tau}{\vartheta_1}\right)e^{-\frac{t}{\vartheta_1}}\right];$$

$$a=a_\infty\left(1-e^{-\frac{t}{\vartheta_1}}\right);\qquad a_\infty=\frac{2\varepsilon(\vartheta_1-\tau)}{(\chi_1-1)\vartheta};\tag{4}$$

$$B=B_p+(B^*-B_p)e^{-\frac{t}{\vartheta_2}};\qquad B^*=\frac{\chi^*G}{\chi G^*};\qquad B_p=\frac{1-\varepsilon\chi B^*}{1+\varepsilon\chi};$$

$$b=b_\infty\left(1-e^{-\frac{t}{\vartheta_2}}\right)+3c;\qquad b_\infty=2\varepsilon\chi\left(\frac{1+\varepsilon\chi^*\frac{G}{G^*}}{1+\varepsilon\chi}-B^*\right);\tag{5}$$

$$C=-B_p-(B^*-B_p)e^{-\frac{t}{\vartheta_2}}+B_pX\left((e^{-\frac{t}{\vartheta_3}}-e^{-\frac{t}{\vartheta_2}}\right);$$

$$X=\frac{1+\frac{\varepsilon}{\chi_1}-\frac{\vartheta}{\vartheta_2}\left(1-\frac{2}{3}\kappa+\frac{\varepsilon\tau}{\chi_1\vartheta}\right)}{1+\frac{\varepsilon}{\chi_1}-\frac{\vartheta}{\vartheta_2}\left(1+\frac{\varepsilon\tau}{\chi_1\vartheta}\right)};$$

$$c=c'\left(e^{-\frac{t}{\vartheta_2}}-e^{-\frac{t}{\vartheta_3}}\right)\qquad c'=\frac{2\varepsilon}{\chi_1}\left(\frac{1+\varepsilon\chi^*\frac{G}{G^*}}{1+\varepsilon\chi}-B^*\right);\tag{6}$$

$$D=\frac{\vartheta_4}{\vartheta}\left[1-\left(1-\frac{\tau}{\vartheta_4}\right)e^{-\frac{t}{\vartheta_4}}\right];$$

$$d = d_{\infty}\left(1 - e^{-\frac{t}{\vartheta_4}}\right) \qquad d_{\infty} = \varepsilon\left(\frac{1+\varepsilon\frac{\tau}{\vartheta}}{1+\varepsilon} - \frac{G}{G^*}\right), \tag{7}$$

where

$$\vartheta_1 = \vartheta\frac{\chi_1 - 1 + 2\varepsilon\frac{\tau}{\vartheta}}{\chi_1 - 1 + 2\varepsilon} \quad \vartheta_2 = \vartheta\frac{1 - 2\kappa + \varepsilon\chi\frac{\tau}{\vartheta}}{1+\varepsilon\chi};$$

$$\vartheta_3 = \vartheta\frac{1+\frac{\varepsilon}{\chi_1}\frac{\tau}{\vartheta}}{1+\frac{\varepsilon}{\chi_1}} \qquad \vartheta_4 = \vartheta\frac{1+\varepsilon\frac{\tau}{\vartheta}}{1+\varepsilon}. \tag{8}$$

These are the required formulae for describing the displacement field.

4. STRAINS IN THE SOLID PROBE

The active element of the probe is an epoxy cylinder of radius ρ with the usual arrangement of strain gauges (**Fig. 2.**). The gauges positioned in the $\theta = 0$ plane measure the radial strains $e_r(0)$ and $e_r(\rho)$ at $r = 0$ and $r = \rho$, respectively. The strain gauges positioned around the circumference of the active element with $\phi = 0$ at $\theta = 0$, $\theta = \frac{\pi}{2}$, $\theta = \frac{5\pi}{4}$ measure the azimuthal strains $e_\theta(0)$, $e_\theta\left(\frac{\pi}{2}\right)$ and $e_\theta\left(\frac{5\pi}{4}\right)$, while those with $\phi = \frac{\pi}{4}$ at positions $\theta = 0$, $\theta = \frac{5\pi}{4}$ measure

$$e(0) = \frac{1}{2}\left[e_\theta(0) + \gamma_{\theta z}(0)\right],$$

$$e\left(\frac{5\pi}{4}\right) = \frac{1}{2}\left[e_\theta\left(\frac{5\pi}{4}\right) + \gamma_{\theta z}\left(\frac{5\pi}{4}\right)\right]$$

giving the value of the engineering shear strains $\gamma_{\theta z}(0)$ and $\gamma_{\theta z}\left(\frac{5\pi}{4}\right)$, respectively.

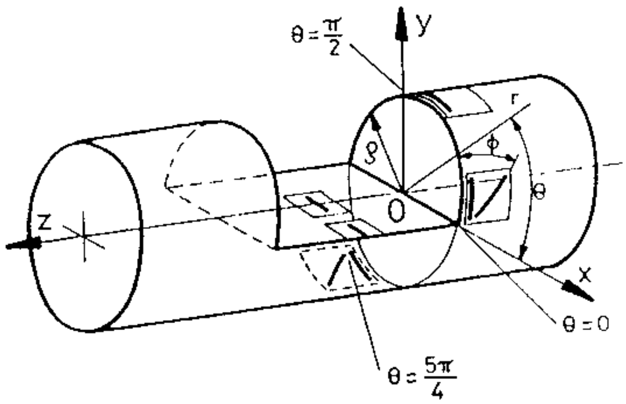

Fig.2. The active element of the probe

The active element of the probe is cast into a solid epoxy cylinder of radius $R < \rho$. The complete instrument is about ten times longer than its strain-gauged section; this is to satisfy the plane strain assumption used in the mathematical solution.

Simultaneously solving for the equilibrium equations of the probe and of the rock mass, the deformation of a solid probe cemented into a borehole can be written as

$$e_r = \frac{1}{2G_1}\left\{\frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)})(\chi_1 - 1)a + \left[b + (\chi_1 - 3)c\left(\frac{r}{R}\right)^2\right]\left[\frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)})\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right]\right\}$$

$$e_\theta = \frac{1}{2G_1}\left\{\frac{1}{2}(\sigma_x^{(0)} + \sigma_y^{(0)})(\chi_1 - 1)a - \left[b - (\chi_1 + 3)c\left(\frac{r}{R}\right)^2\right]\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)})\cos 2\theta + \tau_{xy}^{(0)}\sin 2\theta\right]\right\}$$

$$\gamma_{r\theta} = \frac{1}{G_1}\left\{6c\left(\frac{r}{R}\right)^2 - b\right\}\left[\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)})\sin 2\theta + \tau_{xy}^{(0)}\cos 2\theta\right];$$

$$\gamma_{rz} = \frac{d}{G_1}\left[\tau_{xz}^{(0)}\cos\theta + \tau_{yz}^{(0)}\sin\theta\right];$$

$$\gamma_{\theta z} = \frac{d}{G_1}\left[\tau_{yz}^{(0)}\cos\theta - \tau_{yz}^{(0)}\sin\theta\right];$$

$$\varepsilon_z = 0.$$

5. DETERMINATION OF IN SITU STRESSES

By means of the above formulae one can form at various times the following quantities from the data measured by strain gauges shown in Fig. 2:

$$e_r(\rho) - e_r(0) = \frac{\chi_1 - 3}{2G_1}3c\left(\frac{\rho}{R}\right)^2\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}); \tag{9}$$

$$e_r(\rho) - e_\theta(\rho) - \frac{2(\chi_1 - 1)}{\chi_1 - 3}\left[e_r(\rho) - e_r(0)\right] = \frac{\chi_1 - 1}{2G_1}a\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}); \tag{10}$$

$$e_r(\rho) - e_\theta(\rho) + \frac{4 - \left(\frac{R}{\rho}\right)^2}{\chi_1 - 3}\left[e_r(\rho) - e_r(0)\right] = \frac{1}{2G_1}(b - 3c)\frac{1}{2}(\sigma_x^{(0)} - \sigma_y^{(0)}); \tag{11}$$

$$\gamma_{\theta z}(0) = \frac{d}{G_1}\tau_{yz}^{(0)}. \tag{12}$$

The data given by the left-hand-side of Eq. **10** can be fitted by a function which is similar to **4** in its time dependence. In this numerical procedure the value of parameter ϑ_1 can be determined. By means of Eqs **9, 11** and **12** together with **5, 6** and **7** the time parameters ϑ_2, ϑ_3 and ϑ_4 as well as the time dependent quantities a, b, c, d can be found in the same way.

Measuring at a give time t_1 the strains e_θ and $\gamma_{\theta z}$ at position shown in Fig. 2, the only unknown quantities appearing in the equations:

$$4G_1 e_\theta(0) = (\sigma_x^{(0)} + \sigma_y^{(0)})K_1 - (\sigma_x^{(0)} - \sigma_y^{(0)})K_2;$$

$$4G_1 e_\theta\left(\frac{\pi}{2}\right) = (\sigma_x^{(0)} + \sigma_y^{(0)})K_1 - (\sigma_x^{(0)} - \sigma_y^{(0)})K_2;$$

$$4G_1 e_\theta\left(\frac{5\pi}{4}\right) = (\sigma_x^{(0)} + \sigma_y^{(0)})K_1 - 2\tau_{xy}^{(0)}K_2;$$

$$G_1\gamma_{\theta z}(0) = \tau_{yz}^{(0)}K_3;$$

$$G_1\gamma_{\theta z}\left(\frac{5\pi}{4}\right) = -\tau_{xz}^{(0)}K_3$$

with

$$K_1 = (\chi_1 - 1)a_\infty\left(1 - e^{-\frac{t_1}{\vartheta_1}}\right);$$

$$K_2 = b_\infty\left(1 - e^{-\frac{t_1}{\vartheta_2}}\right) + 3(\chi_1 + 1)\left(\frac{\rho}{R}\right)^2\left(e^{-\frac{t_1}{\vartheta_2}} - e^{-\frac{t_1}{\vartheta_3}}\right)c';$$

$$K_3 = d_\infty\left(1 - e^{-\frac{t_1}{\vartheta_4}}\right)$$

are the in situ stresses. The determinant of the set of equations

$$D = -2K_1K_2^2K_3$$

differs from zero, if $t_1 > 0$ and $\varepsilon \neq 0$. Solving the equations we find

$$\sigma_x^{(0)} = \frac{G_1}{K_1K_2}\left[(K_1 + K_2)e_\theta(0) + (K_2 - K_1)e_\theta\left(\frac{\pi}{2}\right)\right];$$

$$\sigma_y^{(0)} = \frac{G_1}{K_1K_2}\left[(K_1 + K_2)e_\theta(0) + (K_2 - K_1)e_\theta\left(\frac{\pi}{2}\right)\right];$$

$$\tau_{xy}^{(0)} = \frac{G_1}{2K_2}\left[e_\theta\left(\frac{\pi}{2}\right) + \frac{K_1}{K_2}e_\theta(0) - e_\theta\left(\frac{5\pi}{4}\right)\right].$$

These are the required formulae giving the in situ stress components in terms of strains measured by the solid epoxy probe cemented into the borehole. The sixth stress component $\sigma_z^{(0)}$ cannot be measured in this technique, for the probe and the rock mass around its active part are in a state of plane strain.

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